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ABSTRACT

This monograph is designed to furnish classroom teachers with insight and ideas for teaching reading more carefully by emphasizing the what and how aspects of instruction. It is written for the teacher of mathematics in the middle and secondary grades who realizes the important relationship between effective reading skills and learning mathematics. At least one instructional suggestion per page is contained in what is intended as a practical and usable teaching guide. The guide includes such topics as learning to read mathematical symbols, skills for developing vocabulary, analyzing the meaning of words and comprehending the relationships among concepts, and assessing students' abilities. (RB)

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teaching reading and mathematics

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FOREWORD

Mathematics is made up of systems of related ideas or concepts. It is a way of thinking. Through centuries of experience—and careful reflection about that experience—man noted patterns, saw contrasts, and recognized similarities and stabilities embedded in apparent dissimilarities.

Evolving from the systems of ideas we call mathematics, man gradually developed a variety of overt behaviors such as counting, measuring, and computing. Although these procedures prove useful in tackling many day-to-day problems, the essence of mathematics is not merely a set of observable behaviors to be trained into a child. Such behaviors, useful as they are, are only the visible signs of the fundamentals underlying them—the system of ideas. Moreover, these overt procedures are only useful to those who have acquired the concepts underlying them.

In recent years, many educational researchers have focused attention on children and their acquisition of mathematical knowledge. Although differences of opinion exist among these researchers as to how children learn mathematics, and to what extent they ought to learn mathematics, most agree that the mathematical development of children correlates highly with their ability to read mathematics. Further, the more meaningful the approach taken in learning mathematics the more important reading ability becomes.

The relationship between a learner's mathematical development and his or her ability to read mathematics is such that it is necessary for successful mathematics teachers also to be teachers of the reading of mathematics. The importance of this relationship is evidenced by the number of states that now require both preservice and inservice teachers to take coursework in "reading mathematics." However, the degree of success achieved by this training will depend on the quality of the strategies, techniques, models, and ideas presented to and used by teachers. I believe that this monograph, *Teaching Reading and Mathematics*, is an example of a resource that can be used effectively for this type of training. The author not only presents a sound strategy for teaching the reading of mathematics but clearly articulates practical suggestions (by way of examples) for implementing the strategy. That is, he bridges the gap between educational theory and classroom practice.

A. Edward Uprichard, Director
Mathematics Education Clinic
University of South Florida

INTRODUCTION

The Reading Aids Series is designed to furnish classroom teachers with insight and ideas for teaching reading more successfully. This monograph is no exception. It is written for the teacher of mathematics, especially at the middle and upper grades, who realizes (or at least suspects) that success in learning mathematics depends to a large extent on success in reading and for the teacher who realizes (or at least suspects) that successful teaching of mathematical reading is most likely to occur in classes where both the content of mathematics and the process of reading are emphasized equally and constantly.

This monograph intends to provide the what and the how of teaching reading along with mathematics, leaving the why and the wherefore to other, more scholarly, works. If you, as a classroom teacher, do not find at least one instructional suggestion per page, I will have failed in my goal, which is to write a practical and usable teaching guide. Let me hasten to add that the teaching suggestions contained herein are not contradicted by the empirical research available to us. In fact, some of the suggestions are drawn directly from recent research studies. However, the sources of these suggestions are, in large measure, everyday classroom experiences—both my own and those of other teachers of mathematics and reading.

I greatly appreciate the contributions of Carathene Crump and Cleo Embry of Norfolk, Virginia; Caroline Richards of Bloomington, Indiana; Rick Beck of Cincinnati, Ohio; and many others.

RAE

Chapter 1

CONTENT DETERMINES PROCESS

The content of mathematics determines the process necessary for mastery. In other words, the concepts or solutions that make up a reading task in mathematics can reveal to you, the teacher, which reading skills are necessary in order to grasp the concept successfully or to arrive at the correct solution. If you know the goals of your instruction, you can quite easily determine the reading-learning process implied by those goals. Armed with these two essential ingredients—content and process—you will be much better equipped to guide your students to successful reading in mathematics. For your knowledge can then be translated into questions and other teaching devices that reflect a constant awareness of "Where are we going?" and "How do we get there?"—the keys to purposeful and permanent learning.

• Content Analysis

The teaching task actually begins, therefore, with an analysis of the content of a reading task to determine exactly what concepts or solutions are present in the material. How can this be done? First, the teacher identifies the next mathematical topic and selects a short section, chapter, or unit. Then these three steps are followed as he prepares for teaching:

1. Identify the most important concepts or solutions
2. List the words or other symbols essential to each concept or solution
3. Set priorities

Step 1 Identify the most important concepts or solutions

These are the content objectives. The teacher must take the time to write them down so he can refer to them during instruction, making sure he is teaching to his objectives and not to some variation, or different objectives altogether. Although "behavioral" objectives may be used, such statements are not necessary. What is important is that the teacher produce statements of *student understanding*, rather than mere topics or teacher behaviors. Here are some examples of student understandings, stated in a way that gives

the teacher adequate guidance in following the suggestions contained in this volume:

"A set can be specified by describing or by listing the members of that set."

"The perimeter of a polygon is the sum of the length of its sides."

"Mary received $4.96 in change at the store, but lost $.43 on the way home."

Notice how the above differ from the following statements of topic:

"Describing sets"

"Finding perimeter"

"Solving money problems"

Notice also how the above differ from the following statements of teacher behavior:

"To show how sets can be described"

"To define the term *perimeter*"

"To teach word problems involving money"

In order to be certain that their content objectives are stated in terms of student behavior, some teachers have found it useful to ask themselves, "If one of my students studied this assignment successfully, how would he respond?" Or, "What response would I accept from a student as indicating he had in fact understood the concept or solved the problem?"

Step 2 List the words or other symbols essential to each concept or solution

The terms the teacher selects in this step should be integral to each of the objectives. Avoid the temptation to select only "hard" words, such as "commutative," "π," or "hypotenuse." If the terms, "one," "=," or "and" are essential to a stated concept, they must be included along with more difficult or technical terms.

There are several purposes for this. First, if a term is essential to a given understanding, it must be read successfully by *all* the students, not just the "poor readers." Second, listing an "easy" term will help remind the teacher that it bears a critical burden of meaning in this particular reading task. Third, it helps teachers to point out different shades of meaning, or to deepen the student's perception of the precise meaning of particular terms. Fourth, it serves to remind students that in mathematical reading even the easiest word can be critical to an essential understanding.

Step 3 Set priorities

Following the above steps, the teacher has produced written statements of concepts or solutions together with a list of terms judged essential to each. Unfortunately, many teachers, especially at first, are likely to include many more objectives than can possibly be mastered. Therefore, some must be deleted. This is the hardest part of all—the painful process of reducing the number of objectives to manageable proportions. Naturally the actual number will depend on the students, the topic at hand, and time limitations. A rule of thumb (always dangerous) might be three to five concepts or solutions, together with fifteen to thirty key terms. For one whose life is devoted to mathematics teaching, excluding ideas is something akin to infanticide. Nonetheless, the list of content objectives must be held to a reasonable number if teacher or students are to be held accountable for them. *Remember that you can't cover everything!* It may be some consolation to know that the following samples were unflinchingly pared until they represented only the very top priority of the math teacher who prepared them.

Sample Content Analysis No. 1

Reference:	Basic text Chapter 6, Sections 2, 3.
Major Concept:	Ratios can be simplified by treating them as fractions to be restated in simplest terms.
Vocabulary:	ratio fraction simplest common factor : to / divide
Major Concept:	True proportions are proportions whose product of the means equals the product of the extremes.
Vocabulary:	ratio proportion true product means extremes : :: + multiply
Major Concept:	To determine whether a proportion is true or false, one must use the products test.
Vocabulary:	proportion true false products test

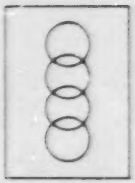
Sample Content Analysis No. 2

Reference:	Basic text Chapter 3, page 125, problem No. 2
Solution:	60 lbs. of water must be evaporated in order to produce a 50% acid solution.
Vocabulary:	solution water acid evaporate 100 pounds x pounds 50 20 %

Caution: Neither mathematics experts nor textbook publishers can be relied on to perform this essential first step—content analysis. It is true that mathematics texts are often well organized and clearly written. In addition, major concepts and important vocabulary are

sometimes identified and listed in the text or teacher's manual. And these aids can help ease the task. However, years of professional training and the particular classroom situation make the classroom teacher the person best equipped to determine exactly what students should learn. "Curriculum," someone once said," is what happens when the classroom door is shut."

This reading specialist does not presume to tell teachers of mathematics *what* they should teach in their classes. However, unless mathematics teachers have analyzed their content, that is, identified the most important concepts or solutions, selected the essential terms, and set priorities, it will be impossible for them to teach either reading or mathematics effectively. This is so because the content objectives determine the necessary reading skills. A description of the reading process follows in the next chapter.



Chapter 2

THE READING PROCESS IN MATHEMATICS

Most professionals agree that reading in any subject matter field cannot and should not be separated from comprehension or concept development in that area. The following is a typical and often used definition of reading by Thomas Barrett, taken from the International Reading Association publication, *The Evaluation of Children's Reading Achievement*:

> Reading involves the visual perception of written symbols and the transformation of the symbols to their explicit or implicit oral counterparts. The oral responses then act as stimuli for a thoughtful reaction on the part of the reader. The type or level of thought induced by the stimuli is determined, in part, by the intent and the background of the reader and the nature of the materials. In addition, the effort expended in the perceptual act and the intellectual impact of the written materials on the reader are influenced by his interest in the specific selection and by his attitude toward reading in general (1968, p. 15).

This definition serves as an abstract base for the classroom model of mathematical reading used throughout the remainder of this monograph. This working model is comprehensive enough to describe the content objectives specified by the teacher, as per the preceding chapter. In addition, it is simple enough to be usable in the classroom by teachers who are not reading specialists. As described by this model, any content objective in mathematics can be mastered by psycholinguistic activity at one or more of the following levels of reading:

1. Perceiving Symbols
2. Attaching Literal Meaning
3. Analyzing Relationships
4. Solving Word Problems

As a general rule, these activities occur in a hierarchy, that is, a reader must be successful at one level of reading before he can hope to achieve success at the next level. For example, if a concept to be gained from reading demands comprehension of ideas stated literally

in the text, the successful reader must first perceive accurately the words and other symbols used to present those ideas. If a particular concept requires an analysis of relationships among literal details, the reader must first perceive the necessary symbols and grasp their literal meaning. Correct solution of a word problem, on the other hand, demands successful reading at all three previous levels. The rest of this chapter describes each level briefly and explains how the nature of your content objectives determines the reading levels necessary for success.

• Perceiving Symbols

For the present purposes, *perceiving* is defined as recognizing and pronouncing; the term *symbols* refers to words essential to mathematical reading, as well as other symbols such as + or =. Hence, the first level of reading in mathematics involves the recognition and pronunciation of any words or other symbols essential to a particular objective. No comprehension is implied at this level; the reader simply recognizes a printed symbol as more or less familiar and pronounces it aloud or to himself. Certainly the concepts and solutions that comprise the content objectives in mathematics cannot be attained solely by activity at this level of reading. Nonetheless, according to our model *any* reading task in mathematics requires success at this level. This is where comprehension really begins. If a reader is unable to pronounce $\sqrt{4}$ as "the square root of 4," it stands to reason that he will have an inordinate amount of difficulty in mastering more sophisticated tasks involving those symbols. The implication for the classroom teacher is clear: He must make every effort to ensure that his students are exposed repeatedly to the sight and sound of *those symbols deemed essential to his content objectives*. Chapter 3 presents a number of suggestions for guidance in perceiving symbols.

• Attaching Literal Meaning

Once the reader has recognized and pronounced the symbols essential to a particular task, he is ready to attach denotative or literal meaning to those symbols. Comprehension at this level depends on two basic elements: symbol meaning and symbol order. Mathematical reading requires the knowledge of a vast number of symbols, many with unfamiliar meanings. The reader must locate and identify symbols integral to a given task and assign meaning to them by sorting out his prior knowledge or by seeking new knowledge. Most of us have mastered the significance of English syntax, that is, the order in which symbols appear, by the time we enter school. Nonetheless, the successful reader must also note carefully the order of symbols in a particular task and seek clarification if the placement of mathematical symbols contradicts his prior knowledge

of syntax. The teacher will find that some content objectives represent understandings stated literally in the text material. His task then is to stress the meanings of those words and other symbols integral to the task and offer aid in locating and verifying these explicitly stated ideas. Chapters 4 and 5 offer a number of suggestions for guidance in attaching literal meaning to individual symbols and larger elements of meaning.

- **Analyzing Relationships**

Nearly any content objective in mathematics requires the reader to grasp several literally stated facts or ideas. Some objectives, however, require a student to identify important *unstated* relationships among those literal facts, and state them in the form of inferences, generalizations, conclusions, or equations. A sophisticated response such as this requires the reader to examine his collection of literal meanings carefully to note common characteristics, facts which do not belong, equalities, and direct or inverse variations. It is at this level of reading that students tend to operate least effectively (perhaps because we teachers tend to teach least effectively at this level). Content objectives which require a response representing meaning implied but not actually stated in the text material will require this level of functioning by students.

Caution: It is easy for the teacher to assume that a "simple" relationship is obvious. Remember that students lack the teacher's wealth of training and experience. If a given content objective is not literally stated in the text, it may be anything but obvious to students. When this level of reading is required, it becomes the role of the teacher to focus the student's attention on the essential literal statements, help note missing or superfluous details, offer guidance in probing and manipulating all possible relationships, and provide hints as to the type of relationships that may apply. Chapter 6 presents several suggestions for guiding students in their analysis of relationships.

- **Solving Word Problems**

Dependent on all three levels of the reading process already described, the solution of word problems is the most sophisticated reading task in mathematics. A major goal of instruction in mathematics is to develop readers who can solve problems successfully and independently. Of course, problem solving can be accomplished nonverbally; indeed, word problems may be an inadequate vehicle for this training. Nonetheless, the solution of word problems represents the most common application of knowledge and skills to intellectual or physical problems—real or simulated—in mathematics.

The reader must perceive the essential symbols accurately, grasp the literal elements, and analyze the relationships correctly before he can arrive at a correct or creative solution to the problem. With these prerequisites accomplished, the solution to the problem may be relatively routine; without them, the process is marked by total frustration, trial and error, or only ignorant success. Good teaching involves necessary guidance at all preceding levels of the process. The teacher must keep the student aware of the total reading process by continuing to point him in the right direction without actually providing the solution. Guidance at this level will lead the student to reading success; that is, he will be able to reconstruct the problem in the form of an algorithm, at which time logic or computation skills can be called on to complete the task. Chapter 7 presents suggestions for guiding students in solving word problems.

- **Sample Questions**

The following guiding questions were formulated to help students master the concepts listed in Sample Content Analysis No. 1, outlined in Chapter 1. *It is important to note that the teacher did not set out to ask questions at each of the levels.* Since the substance of the content objectives determined the necessary process level, the questions illuminate that process as they lead the student to mastery of the task.

Ratio and Proportion (Basic Text, pp. 123–125)

Perceiving Symbols

1. Observing carefully the *sight* and *sound* of the key terms in the left-hand column, match each term with the best description from the right-hand column. Work with your partner. You *must* reach agreement.

_____ ratio	a. is to
_____ extreme	b. three syllables
_____ proportion	c. Horatio
_____ products test	d. plural
_____ :	e. rhymes with cream
_____ means	f. two separate words

Attaching Literal Meaning

2. With reference to Chapter 6, Sections 2-3 of your basic text (pages 123–125) verify or reject each of the following. Use "V" for verification, "R" for rejection. You must be able to prove your choices by reading from the text.

_____ A ratio is the comparison of one number to another by division.

_____ A proportion consists of two equal ratios.

_____ The symbol : : indicates an equality of ratios.

_____ The best way to determine true proportions is to use the products test.

_____ The term *extremes* means the two greatest numbers in a proportion.

Analyzing Relationships

3. Based on your study of ratio and proportion thus far, which of these statements do you think are true? You *must* explain your decisions.

_____ 1/15 represents a ratio.

_____ ratio: proportion : : dimension: volume

_____ 2/3 = 4/6 is a false proportion.

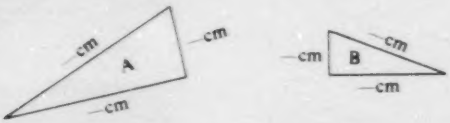
_____ The products test is really only cross multiplication.

_____ For true proportions, the means equal the extremes, with *one* exception.

_____ Means are always numerators; extremes are always denominators.

Solving Word Problems

4. Are these two triangles proportional?



Hint: _____ : _____ : : _____ : _____

You must show your work in order to explain your thinking and prove your solution.

The above examples suggest that questions can be formulated to guide students in the reading of mathematics, as determined by the teacher's stated content objectives.

- **Role of the Content Analysis**

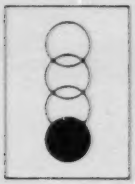
It is, of course, possible to teach the process described above for its own sake; special reading teachers do it all the time. However, for the mathematics teacher, reading instruction will be more purposeful and efficient if it is confined to those skills or levels of the process necessitated by the content objectives judged essential to a particular text assignment or unit, as suggested in Chapter 1. At the risk of being redundant, it is absolutely essential to remember that only the mathematical task itself, that is, the *content objective*, can determine which levels of the reading process are necessary for student mastery.

- **Summary**

The model of reading presented here is useful in explaining the means by which we all process language, both spoken and written.

It certainly describes the psycholinguistic *how* of nearly any substantive task in mathematics: expository passages, graphs, word problems, and charts. It is presented as a usable instructional model for reading and learning. A large number of specific skills, often suggested as important to mathematical reading, have been omitted, since nearly all are encompassed by the model at an appropriate level when demanded by a particular reading task, that is, by a particular content objective. Study techniques, such as SQ3R, directed at the student are excluded as well. The intent here has been to provide the teacher with a workable definition of mathematical reading, and to explain how this knowledge can be useful in guiding students in the classroom.

The type and amount of guidance required will vary with individual students, depending on the specific reading task involved. In fact, an assessment of students' reading abilities, as explained in Chapter 9, may indicate that guidance is unnecessary at some levels of the process for certain objectives. Students who can operate independently do not require guidance. For example, some word problems can be solved readily by all the students in a given classroom. On the other hand, grasping literal meaning is sometimes impossible for the majority of students, given a particular text assignment. It is easy, then, to assume that all students will (or should) be successful. We rarely seem to err at the other extreme, by providing too much guidance in reading. As noted earlier, the following chapters are designed to furnish a number of specific suggestions for providing guidance at each of these levels.



Chapter 3

PERCEIVING SYMBOLS

The accurate recognition and pronunciation of symbols—both words and other terms—are the first steps in successful mathematical reading. If each of your students can recognize instantly and pronounce accurately all symbols essential to a particular reading task, there is no need for further guidance at this level of the reading process. This happy condition seldom prevails; therefore, this chapter offers a number of sample exercises and suggestions which should improve your students' skills in functioning at this level.

• Role of the Content Analysis

Of course we wish our students to be able to perceive every symbol accurately, and that is our ultimate goal. However, there is an element of redundancy in all text material—even the concise writing of mathematics—which makes this unnecessary in most reading assignments. More important, the task must be held to manageable proportions, so both you and your students will know when the goal is reached. This is the reason for setting priorities as part of your content analysis. The content analysis yields a relatively small number of symbols judged essential to the reading task. It is these symbols, therefore, which are stressed in exercises designed to improve students' facilities in perceiving symbols.

• Paper and Pencil Exercises

Averages

Key Words Math 9

Directions: This exercise is designed as a race against yourself in spotting key words quickly and accurately. Look at the first word in each row. Then circle that word *every time* it appears in that row. When you are finished, record your time in seconds in the space provided.

1.	average	average	averaged	averages	average	
2.	mean	meat	clean	near	bean	
3.	typical	typcal	typist	typikal	critical	
4.	median	media	median	meridian	mode	_____seconds

This particular exercise contained 17 rows of words, each row beginning with a key symbol selected by the teacher during the content analysis. It consumed a total of about five minutes of class time, with the teacher confirming the directions, then recording elapsed time in seconds on the chalkboard (5, 10, 15) as the students completed the task. As each student finished, he recorded the last figure appearing on the chalkboard, which amount of time became his score. The exercise was used three times in this class of relatively unsuccessful readers: at the beginning, middle, and end of the unit. At each administration, the teacher encouraged her students to improve their speed and accuracy. And they did!

This type of exercise permits a number of variations in construction and administration. If serious perception problems plague your class, adjust the exercise to include more obvious discriminations. For example,

1.	average	fish	John	average	grandmother	
1.	average	xyzage	averxyz	avxyzge	average	_____seconds

The exercise can also be designed so that students are directed to circle any symbol that appears on a certain page or in a particular section of their text material. Sometimes it is useful to allow students a practice run (minus pencil), touching the first term in the row and all other matching terms, and recording their time in seconds. The next time through is "for the record," the object being to improve their accuracy and shave a few seconds off their initial scores.

Ratio and Proportion

Directions: It is important to read mathematics accurately. Below you will find several statements taken from your text. The exact page and paragraph number follow each statement. Your task is to examine each statement carefully to determine if it is *exactly* the way it appears in the text. If it is, label it OK. If it is not, change it to produce an exact duplicate. Record how much time it takes you to do this.

1. The perimeter of a polygon is defined as the sum of its sides (p. 122, par. 2).
2. In other words, the perimeter is the distance around a polygon (p. 122, par. 2).
3. We are going to study finding perimeter by using formulas (p. 123, par. 1).

_____Ending Time
_____Starting Time
_____Time Spent

This type of exercise places more emphasis on accurate perception of symbols in the text itself. Needless to say, the sentences selected by the teacher are those which contain a high percentage of symbols judged to be essential to the mastery of the content objectives.

Radicals

Directions: Working with your partner, decide whether you agree (A) or disagree (D) with each of the first three statements. Then complete the rest of the statements correctly.

1. "$\sqrt{\ }$" is read "the second root of."
2. "$\sqrt[3]{18}$" is read "the cube root of eighteen."
3. "$\sqrt[4]{16}$" is read "the fourth root of sixteen."
4. "$\sqrt{10}$" is read "_____."
5. "_____" is read "the square root of fifty-six."

Algebraic Statements

Directions: Match the algebraic statement on the left with the translation on the right. Collaborate with your work group. *Everyone must agree.*

__1. ab	a. the square of a number
__2. x^2	b. the square of the sum of two numbers equals the square of the first number, plus twice the product of the numbers, plus the square of the second number
__3. $x^2 - y^2$	c. the product of two numbers
__4. $(f + s)^2 = f^2 + 2fs + s^2$	d. the difference of the squares of two numbers

Notice that some of these examples require two or more students to work together. This encourages students to complete the exercises orally, offering an opportunity for repeated experience with the sight and sound of the key symbols.

- **Overhead Projector**

Print a number of symbols randomly on a blank sheet of acetate. Include some terms that are not related to the particular topic at hand. Assign your students to teams of two, one designated as the spotter and one as the recorder. When the projector is turned on, the spotter pronounces (and spells, if necessary) all symbols he remembers seeing and hearing during the study of a particular unit, chapter, or reading assignment. The recorder makes a list of the words identified by the spotter. The spotter is not allowed to check the recorder's paper; the recorder is not allowed to check the sym-

bols being projected on the screen or wall. Teams can be scored by counting one point for each key term identified, minus two points for each irrelevant term listed. This game has proved interesting to students with word perception problems, even at the high school level. More importantly, it requires all the participants to practice the accurate perception of the essential symbols in a unit of study. The overhead projector is also useful in presenting some of the exercises described earlier.

- **Chalkboard**

The use of the chalkboard in furthering accurate perception is all too frequently overlooked. When introducing key terms, or when an important symbol arises in classroom discussion, write or print it on the chalkboard, or request that a student do so. The correct pronunciation can be elicited from the entire class and reinforced by the teacher or individual students. A variation of this technique is to print a symbol on the board and ask students to raise their hands when they find the term printed on a certain page in the text material. Or ask each student to record on scrap paper the *number of times* a certain symbol appears on a certain page. A brisk walk around the room to check student responses provides insight as to student perception and is good exercise for the teacher as well.

- **Labeling**

The use of labels in the classroom should not be confined to kindergarten coat racks. Mathematical labels essential to the objectives can be constructed by either students or teacher, or cut from newspapers, magazines, or old textbooks or workbooks. These labels can be attached to pictures, drawings, designs, and artifacts. As a matter of fact, labels representing key symbols can be posted in the classroom for their own sake—on bulletin boards, walls, windows, ceilings, floors, and on the desks of students and teacher.

Another interesting use of labels is to assign each student a mathematical middle name which he can wear throughout a given unit of study. This creates interest and keeps students exposed to the correct pronunciation of important terms.

- **Flash Cards**

Symbols essential to a given topic can be printed (again, by either students or teacher) on small cards and used productively for practice by pairs of students who have difficulty at the perception level. Although 3 × 5 cards can be cut and utilized for this purpose, unused play tickets or extra ducats for athletic events are often available. Check with the main office, the drama department, or the coach. One student in the assigned pair reads the terms aloud as

rapidly and accurately as he can, while the other checks his accuracy and time. Then the roles are reversed. In a few minutes, the students will have helped each other gain valuable familiarity with the symbols.

• Flash Devices

A number of mechanical devices are available which are useful in providing practice in perceiving symbols. Some, such as the Controlled Reader (Educational Developmental Laboratories, 284 Pulaski Road, Huntington, New York 11743), are sophisticated and expensive. Others are much cheaper. For example, if you have access to an ordinary filmstrip or slide projector, you can purchase inexpensive frosted filmstrip (Hudson Photographic Industries, Irvington-on-Hudson, New York 10533) or frosted glass slides (Society for Visual Learning, 1345 Diversey Parkway, Chicago, Illinois 60614). Mathematical symbols can be printed on filmstrip or slide with ordinary pen or pencil, and can be erased and reused many times. For rapid recognition practice, a device such as the Tachist-O-Flasher (Learning through Seeing, Sunland, California 91040) or the Speed-i-o-graph (Graflex Company, 3750 Monroe Avenue, Rochester, New York 14603) is placed before the projector. Pressing a shutter on either of these devices exposes students to the sight of the symbol for about 1/25 of a second, enough time for the good reader to recognize it. The Flash-X (also EDL) is an inexpensive hand-held device of similar nature.

The cheapest form of flash device is the hand tachistoscope, which can be fashioned from cardboard and used with typed lists of symbols integral to a topic of study. Appendix A contains directions for constructing a hand tachistoscope. This sort of practice can be provided for whole classes, although teachers find it most useful with individuals or small groups who are not adept at perceiving symbols. Students really enjoy this type of exercise.

• The Language Master

Another device often available in schools or school districts is the Language Master (Bell and Howell, 7100 McCormick Road, Chicago, Illinois 60645). Although designed for use in reading clinics, the Language Master can be very helpful in the mathematics classroom as well. This machine employs large cards to which strips of recording tape have been attached. Using symbols derived from the content analysis, the cards are prepared by the teacher or a willing student. Each symbol is printed on a card and its correct pronunciation recorded on the tape. The individual student who needs practice in perceiving symbols studies each term, records his pronunciation, then throws a switch which enables him to check the correct

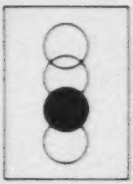
pronunciation recorded earlier. Extra-class practice periods are usually unnecessary. A few minutes at the beginning or end of the regular class can bring about a big improvement.

- **Word Lists**

It is common practice to suggest that students compile lists of unfamiliar words which arise in mathematics. Certainly, this is useful on occasion. Unfortunately, recording a list of symbols that are truly unfamiliar is a very difficult task for students who are not already relatively independent at this level. In addition, making a list of words can become a deadly dull chore. Researchers have compiled lists of words most frequently used in mathematics text material. For example, one such list includes the 473 words most frequently appearing in eight widely used basal arithmetic series (Appendix B). Lists such as these can serve many useful purposes. However, avoid the temptation to "overkill." Students become easily discouraged when they are asked to practice their perception on seemingly endless lists of symbols. It is more advisable to use finite lists selected on the basis of a careful content analysis, as recommended in Chapter 1. This will make the guiding activities suggested here more purposeful and more manageable. Both teacher and students will know what the task is and when it is finished.

- **Summary**

The more times students are exposed to the sight and sound of important mathematical symbols, the more likely they are to recognize those symbols accurately and rapidly. Guiding activities suggested here can be varied in an almost infinite number of ways, depending on the particular needs of a given class. With a little thought, other activities can be devised as well. Mathematical reading begins with the efficient perception of essential symbols. Efforts here can help prepare students for the next level of the reading process, which is the topic of the following chapter.



Chapter 4

ATTACHING LITERAL MEANING: VOCABULARY

If your students are able to recognize and pronounce the essential words and other symbols in a particular mathematics unit or assignment, they are ready to proceed to the next higher level of mathematical reading—attaching literal meaning to individual symbols and groups of symbols. For the moment, let us concentrate on individual symbols.

Teachers of mathematics certainly realize the importance of a large vocabulary to success in their subject. Inaccurate or imprecise definitions of essential terms make successful reading or learning difficult or impossible. Therefore, most teachers feel an obligation to introduce and reinforce on a continuing basis the meanings of those symbols they regard as essential to their content objectives. If all students demonstrate uniformly high levels of vocabulary mastery, no guidance is necessary here. However, in the likely event that your students need help in defining important terms, this chapter offers several suggestions.

• Vocabulary Types

Most mathematics teachers are interested to learn of three commonly recognized types of vocabulary: general, technical, and special. General vocabulary is just that, words or symbols used regularly in all walks of life on a daily basis. Examples of general vocabulary are terms like *chair*, *love*, *$*, *closed*, and *property*. Technical vocabulary, on the other hand, are terms peculiar to a certain field of study or area of interest. In mathematics, terms such as *polygon*, *hypotenuse*, $\neq$, or *quotient* could be considered technical terms. But the most difficult task of all for the student is posed by special vocabulary, symbols which have one meaning in everyday life but take on a different, or specialized, meaning in the context of mathematics. Examples of special mathematics vocabulary are *point*, *base*, *closed*, and *property*.

• Defining Terms

All vocabulary terms, including those in mathematics, can be defined in several ways:

1. Formal definition

   Example: A triangle is a figure formed by three distinct line segments, each pair of segments having exactly one point in common.

2. Listing of characteristics

   Example: Several things are true about any triangle. It has only two dimensions, length and height. It has exactly three sides, each one a straight line. It has three interior angles, which total exactly 180 degrees.

3. Simulated examples

   Example: Triangles drawn on a chalkboard, cut from paper material, or pointed out in drawings or pictures.

4. Real life examples

   Example: Natural occurrences which exemplify triangular formation. Manmade objects which utilize triangles in industrial, architectural, or decorative features.

All four means of definition listed above can and should be used with students. As a general rule, however, teachers find the above means are listed in order of increasing effectiveness. However, simulated and real life examples of mathematical concepts are not always available to us in easily explainable form. Hence our reliance on the more abstract means of definition. The teaching suggestions in this chapter, therefore, emphasize the use of written symbols.

- **Glossing**

Many teachers, who have asked students to look up and write definitions of mathematical terms, are disappointed when their students copy definitions verbatim from glossary or dictionary and then prove by subsequent work that they do not have a firm grasp of the meanings. Formal definitions, as suggested earlier, may not make sense to your students without a good deal of additional guidance from the teacher. Therefore, some teachers have found that glossing—that is, having students prepare their own glossary of terms—is a useful activity. In glossing, a small group of students is given a few mathematical symbols essential to the topic at hand. The students are asked to write their own definitions, using language that makes sense to them. The resulting definitions are not as elegant as dictionary definitions but prove considerably more useful. At the end of the activity, after all members of the team agree on the definition, the cards can be filed alphabetically for reference by other students. As with most such practices, however, the real benefit is derived by those who do the glossing.

- **Crossword Puzzle**

Another means of stressing definition is the simple crossword puzzle. Although the usual newspaper-type crossword can be constructed, the arrangement of the correct combination of across and down terms is usually more than enough to try the teacher's patience and almost always consumes more time than teachers have to spend. Therefore, the following type of crossword is a good compromise, combining ease of construction with student appeal.

Directions: Using the hints provided below, complete the spelling of each of the following key words. Do *only* those you are certain you know—you'll learn the meanings of the rest in the next few days as we continue to study this topic.

1. _ _ C _ _ _ _
2. _ I _ _ _ _
3. _ _ R _ _ _ _ _ _
4. C _ _ _ _ _ _ _ _ _ _ _
5. _ _ _ _ _ _ L _
6. _ E _

Hints:

1. the numbers to be multiplied
2. the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
3. do the operation within these first
4. the sum is the same in either order
5. symbols used for the numbers
6. used as a grouping number or base

Notice that the directions do *not* encourage guessing. The teacher in this instance used the crossword as a combination pretest and learning exercise. As instruction progressed, every child in class was able to complete the puzzle by listening, reading or consulting with peers or teacher. Simple record keeping made it possible for every child to see his progress.

- **Multiple Characteristics**

Guiding students in the selection of appropriate characteristics can be very useful in defining symbols, perhaps because such listings tend to employ language less formal than that found in glossary definitions. The following example is an attempt to focus students on several important attributes of closed figures.

Directions: Check each characteristic that helps to describe the underlined terms below. Be careful! Each term may have several characteristics.

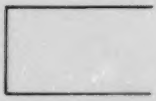
1. a simple closed figure
   - ____ is drawn in a plane
   - ____ has boundaries that intersect
   - ____ is a polygon
   - ____ encloses part of a plane
   - ____ is formed by line segments
   - ____ always has four or more sides

2. a polygon
   - ____ is a simple closed figure
   - ____ is formed by a line segment
   - ____ is not drawn on a plane
   - ____ encloses part of a plane
   - ____ is like a circle

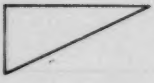
3. lw
   - ____ is a misprint
   - ____ is a formula
   - ____ can be applied to all polygons
   - ____ stands for length and width
   - ____ should be printed in capital letters
   - ____ indicates multiplication

The following is one of several possible variations in listing or describing characteristics of important mathematical terms.

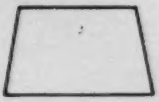
Directions: Circle the word(s) at the right which best describe the figure at the left.

1. 
   a. a simple closed figure b. polygon c. neither

2.
   a. a simple closed figure b. polygon c. neither

3. 
   a. a simple closed figure b. polygon c. neither

4.
   a. a simple closed figure b. polygon c. neither

5. 

a. a simple closed figure b. polygon
c. neither

- **Multiple Meanings and Multiple Terms**

Even in the concise language of mathematics, a single term can often have many meanings. The following is an example of an exercise designed to guide students in selecting multiple meanings.

Directions: You are to combine phrases with the underlined words in each section to make true statements. Indicate your choice of phrases by placing a check on the line before the phrase. You *may* check more than one phrase in a section. Be prepared to support your choices.

1. a number
   - ____ is like a numeral
   - ____ is indicated by some symbol
   - ____ is a placeholder
   - ____ is an idea
   - ____ can be written on the chalkboard
2. zero
   - ____ is a counting number
   - ____ is a real number
   - ____ is a numeral
   - ____ is a placeholder
   - ____ is an element of the null set

The opposite side of this coin is a single definition which fits several important terms used in mathematics. The following exercise guides students in identifying multiple terms.

Directions: Check each term that is at least partly described by the underlined meaning.

1. Can be considered an average
   - ____ number ____ mode
   - ____ median ____ mean
   - ____ triangle ____ quotient
2. Is part of a number
   - ____ zero ____ digit
   - ____ factor ____ element
   - ____ numeral ____ ϕ

- **Context Analysis**

Another useful approach in defining terms is *contextual analysis*—the process of clarifying meaning by examining the mathematical context in which a term appears. Students become more aware of the importance of context in determining meaning when the teacher presents several alternatives, using the chalkboard or overhead projector. For example, one teacher listed these sentences on the chalkboard, asking, "In which of these sentences is the word *simple* used in the same way as in your text on page 23?"

That problem is *simple*.

Simple Simon was not good in math.

Betty's skates traced a *simple* curve in the ice.

Another approach, which provides more thinking time for all students in a class, is to change the mathematical term to a nonsense word and ask students to provide all possible words that make sense in the particular context. For example:

The police line formed a *dulimp* area around the president.

Our disagreement last week is a *dulimp* matter.

When I arrived, the store was *dulimp*.

As the students are shown each sentence, they are encouraged to speculate on all possible sensible words to replace the nonsense word, *dulimp*. The teacher lists all contributions on the chalkboard or overhead projector, then directs students to a specific page in the text material to confirm their guesses.

The simplest type of context clue is the straightforward definition in text material—an unfamiliar word followed by *is* or *means*. The teacher will take care to point out such obvious facts to students. Of course, some context clues are not recognized as easily, especially when special vocabulary is involved. In that case students often benefit from exercises such as the following:

Special Vocabulary

Directions: Listed below are words or phrases necessary to our study of Chapter 2. Some of the words or phrases have different meanings used in other subjects. Some have the same meanings when used in mathematics as when used in other subjects. Draw a line under those words or phrases which have a *special meaning when used in mathematics*. Page references are in parentheses.

symbol (22)	one-to-one matching (43)
numeral (23)	tally (22)
place value (22)	exponent (50)
expanded numeral (50)	set (23)

elements (30)	digit (22)
base (40)	binary (32)

An exercise similar to that shown above can help to further the distinction between general and special uses of symbols in mathematical context. Other written exercises, which can be presented on paper, or by means of the chalkboard or overhead projector, include the following:

Scrambled Word

Directions: Unscramble each key term to fit its definition.

1. When you add all the data and divide by the number of addends, the result is called the NEMA.
2. The mode is the value that appears in your data most QULFYRETNE.

Some teachers object to presenting scrambled words, in the belief that it encourages poor spelling. Therefore, at the teacher's discretion, the scrambled words can be replaced by blanks of standard length, as in the following. Note that in this case, the teacher aided the students in their selection by providing a word list (Set II).

Completion

Directions: Fill in the spaces in Set I by selecting the correct word(s) from Set II.

Set I

1. The boundary plus interior of a simple closed figure is called a _____.
2. Simple closed figures are called _____.
3. The set of points outside a simple closed figure is called the _____.
4. If a closed figure does not cross itself it is called a _____.
5. The set of points inside a closed figure is called the _____.
6. A line or curve from Point A to Point B and back to Point A by another path is called a _____.

Set II

closed figure	interior
region	simple closed curves
simple closed figure	exterior

- **Matching**

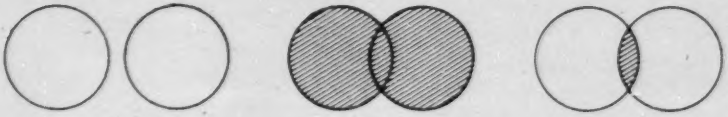
In the context of a particular mathematical topic, the common matching exercise can be useful. Note the two types of matching exercises which follow.

Directions: Match the definitions in the second column with the words they define in the first column. Write the letter of the definition to the left of the word defined.

COLUMN 1	COLUMN 2
____ set	A. list of members
____ natural numbers	B. a collection of numbers
____ natural numbers and zero	C. counting numbers
____ roster	D. whole numbers

Directions: Under each diagram write the term that fits it best.

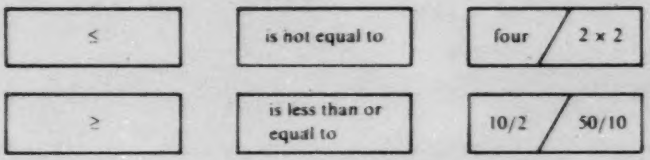
intersection of sets
disjoint sets
union of sets



Another variation on the matching exercise is the following game which combines symbol perception with definition.

Directions: These eighteen cards can be divided into six groups of three each, with each group containing one mathematical symbol, its correct pronunciation, and an example of two values which illustrate the symbol's meaning. Work with a partner and try your luck. When you finish, check with your teacher or another student. Keep a record of the time it takes you to do this.

<	equals	twelve / twenty
=	is greater than or equal to	5¢ / 1¢
>	is less than	10 / $10.00
≠	is greater than	six / ten



- **Structural Analysis**

An analysis of structural clues to word meaning involves the process of identifying familiar or meaningful parts of unfamiliar terms. Structural analysis is used by skilled readers to attach partial meaning to important terms—perhaps enough to clear away a little confusion and add a bit of meaning to an initial exposure to a key symbol. Teachers like to point out structural clues to their students to make them more sensitive to frequently used prefixes, suffixes, and word roots in mathematics. Structural clues rarely define a word completely; however; they do encourage students to draw on their own knowledge of word parts often used in mathematics. Many mathematical terms lend themselves to structural analysis: *parallelogram, triangle, quadrilateral, equilateral, monomial, polynomial, tetrahedron*, and others. The best time to encourage student use of structural analysis is whenever such terms come up in the classroom. For example, "Here's a word we'll be dealing with a lot in the next few days. Who can pronounce it? What does it mean? Does any *part* of the word look familiar to you? What other words have we studied that use that word part?"

Examination of vocabulary selected as essential for a given topic can result in a list of word parts worth pointing out to students. Some teachers actually design exercises which require students to build mathematical terms—real or imaginary—from the parts supplied. For example, one such list might include some or all of the following.

ante-... before
anti-... against
auto-... self
bi-... two
circum-... around
inter-... between
mis-... wrong
non-... not
poly-... many
post-... after
semi-... half
super-... above
trans-... across

½... hemi-, demi-
1... uni-, mono-
2... bi-, di-
3... tri-
4... quadra-, tetra-
5... penta-, quin-
6... hex-, hexa-
7... sept-, hepta-
8... oct-
9... nona-
10... dec-
100... centri-

Another important aspect of structural analysis is illustrated by the following exercise used by a teacher to help her students pick the mathematical base word for the vocabulary essential to a short unit of instruction:

Directions: Draw a box around the mathematical base word in each term below. The first one has been done for you. When you finish, compare your choices with your nearest neighbor, and try to iron out any differences.

[inch]	es	tenth	estimating
measurement		measuring	nonmetric
foot		meterstick	postcheck

Another interesting means of emphasizing the structural elements of essential terms is to provide each student with a card on which is printed (by the students) a word part (root, base word, prefix, suffix, inflectional ending) important to the topic at hand. At a signal from the teacher, the students can form groups which make larger or derived forms of words by combining the various elements. This is fun, but it is more than mere recreation. Such practice sensitizes students to the various combinations of mathematical vocabulary, and illuminates the changes in meaning which result from different combinations.

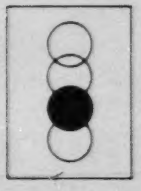
It is worth noting that little will be gained by attempting to teach word parts formally apart from key terms immediately under study in the classroom. The proper time to emphasize the structure of vocabulary is at the exact time it is integral to a topic of study.

When students have attacked a word or symbol through structural analysis, a moment spent confirming their hunches in a dictionary or glossary is a moment well spent reinforcing the meaning, correcting misconceptions, and, perhaps most important, acquiring the idea that a glossary or dictionary may be useful after all—and painless.

• Summary

All the activities suggested in this chapter are especially helpful when students are required to share and defend their understandings, either collaborating on the task itself or consulting with each other after completing the exercises individually. Research in the area of vocabulary development suggests strongly that there is no one best way to ensure optimum mastery of all important terms. The suggestions presented in this chapter, therefore, suggest a wide variety of techniques which the teacher can use in the mathematics classroom to emphasize essential terms. And we *do* know that constant emphasis of almost any kind results in increased student mastery of mathematics vocabulary. Given the hundreds of important mathe-

matical terms, moreover, both teacher and student will find the task more manageable and comfortable if relatively few symbols are selected for emphasis at any one time. Finally, the activities suggested here have two important common factors: 1) they include only those symbols judged by the teacher to be essential to the topic, chapter, or unit under study; and 2) they do not assume that students have mastered the meanings simply because they can pronounce the terms readily or because "they have had them before." The following chapter offers suggestions for helping students grasp larger (but still literal) elements of meaning presented in text material.



Chapter 5

ATTACHING LITERAL MEANING: EXPLICIT IDEAS

When the teacher's content objectives require the student to grasp larger elements of literal meaning, that is, ideas or understandings stated explicitly in text material, guided reading is an effective instructional technique. Guided reading is directed toward the dual purpose of helping the students master essential mathematical *ideas* by successful use of the reading-learning *process*. At this stage, the teacher helps the students locate and verify ideas presented explicitly in the required text material. This chapter offers suggestions for guiding your students in grasping those ideas—essential to the content objectives—stated literally in text assignments.

• Purpose Questions

We would like to believe that each student has attained enough sophistication to set his own purposes for reading the material required at his particular instructional level. Unfortunately, this sort of sophistication develops slowly, and at a different rate for each student. And, without a specific purpose, the unsophisticated reader has two alternatives. On the one hand, he can attempt to realize complete mastery of a reading assignment, a goal which is not realistic for most students, even given a short assignment. The second alternative is to give up in frustration, hoping the teacher will repeat the important points in class. With either alternative, both content mastery and continued growth in reading ability suffer from delay or total stoppage. Many teachers, therefore, find that they can focus their students on the essential understandings *if* they first have performed a content analysis as described in Chapter 1. A student who is looking for nothing in a reading assignment usually finds it. A student who knows why he is reading (for example, to locate and list the steps necessary in finding the perimeter of a rectangle) is much more likely to be successful. The following are examples of purpose questions, which can be presented by the teacher orally, or by using the chalkboard, overhead projector, or ditto master.

- What two things are number lines used for?
- According to your text, area is always measured in what kind of units?

- How does your author describe a plane coordinate system? What does a plane coordinate system enable you to do? (P. 253, Par. 3–4)
- In what two ways can you specify a set? Which is the best way, according to your author?
- Describe the products test. When would you use it? (Chap. 12, Sec. 5)

Note that in some of the above examples the teacher has supplied not only the assignment but the exact page and paragraph as well. This indicates that the teacher realizes the value of guided reading and is willing to provide whatever guidance is necessary to help the students achieve success in reading.

If teachers expect their students to read with definite questions in mind, and are willing to supply those purposes if necessary, the probability increases that reading assignments will be more valuable and rewarding to everyone concerned.

- **Study Guides**

A study guide is another way to present questions for the purpose of helping students achieve reading success. Like purpose questions, study guides focus students on those understandings which are essential to the content objectives. However, they offer additional elements of guidance; for example, locational information, possible alternatives to verify from the text, or opportunities to actively manipulate information gained from the reading assignment. Examine the following study guides, all of which seek to guide students in the successful acquisition of explicitly stated ideas deemed essential to the mathematical objectives.

Sample Study Guide #1

Multiplication Facts

Directions: Check the statements that are true, *using your Multiplication Chart.* (Page 341) Correct the statements that are not true.

____7 × 3 = 21	____2 × 6 = 12
____6 × 2 = 12	____8 × 7 = 72
____three times two equals six	____one times seven equals seven
____four times three equals eight	____0 = 2 × 0
____five times nine equals twenty-seven	

Sample Study Guide #2

Simple Closed Figures

Directions: Read the first statement and then the lines (on page 153) in the text to which you are directed. Decide if the author says what

the statement says. If you think so, place a check on the line before the statement. If not, leave the line blank. Do this for each statement. After you have completed this part, compare your answers with others in your group. *Where you have differences, try to resolve them.*

____1. Simple closed figures are also called simple closed curves. (4)

____2. The set of points outside the figure is called the exterior. (11, 12)

____3. A polygon is a simple closed figure. (15)

____4. Polygons have adjacent sides. (17, 18)

____5. Some polygons have diagonals. (25, 26)

____6. The boundary of a closed figure is the figure itself (9–12)

____7. Polygons have one region. (12, 13)

____8. A triangle is a three-sided polygon. (20, 21)

Sample Study Guide #3

The Number Line

Directions: On page 25, paragraph 2 in your text, the author explains the Number Line. Which of these statements are *literally stated* by the author?

____On the number line, all points to the left of zero are positive.

____ -5 is an example of a number that is to the left of zero on the number line.

____On the number line, any number is less than every number to its right.

Sample Study Guide #4

Pythagorean Theorem

Directions: The following passage was taken directly from your text. Every fifth word has been deleted. *Without looking at your text* see how many sensible words you can replace. Hint: Sometimes the word that makes the most sense is a very simple word, rather than a "mathematics" word. You may be surprised at how many words you can supply that make the completed passage sensible reading. Feel free to share your ideas with your working partner, and be sure to ask him for his ideas. When you have completed as much of the "message" as you can, refer to page 242 in your text, find out how well you did, and complete the message so the author would approve of it.

The theorem to be proved in this section is one of the best known and most useful theorems in all mathematics. It states an _____ relationship between the lengths _____ the legs and the _____ of the hypotenuse of _____ right triangle. Most of _____ are already familiar with _____ relationship, perhaps knowing it _____

such names as the "_____ Triangle Principle," the "Pythagorean _____," or the "Rule of _____."

In any right triangle _____ square of the length _____ the hypotenuse is equal _____ the sum of the _____ of the lengths of _____ legs.

From Jurgensen, Donnelly, and Dolciani, *Modern School Mathematics: Geometry*, Chapter 9. Boston: Houghton Mifflin, 1967.

Sample Study Guide #5

Word Problems

Directions: In problem #4 (page 68) which of the following information is given to you about the Browns' dessert? *You must be able to prove your choices by reading from the text.*

____ The Browns had cake for dessert.

____ They ate the whole dessert.

____ They ate 2/3 of the dessert.

____ They had only 3/4 of a dessert to start.

____ They ate three-fourths of it.

____ They started with two-thirds of a pie.

____ Dinner lasted three-quarters of an hour.

The reader will note that the directions for several of the above study guides direct the student to compare his work with one or more of his peers, defend his responses, listen to other points of view, and resolve any differences. This provides an active involvement in the reading process, including much verbalization using the language of mathematics and frequent reference to the assigned text material. Although it is sometimes frightening to contemplate at first, many teachers find that short periods of such group discussion increase their students' interest as well as verbal ability in mathematics.

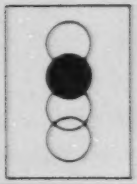
- **Role of the Content Analysis**

The literal level of reading, where tangible meaning begins, is called into play in order to acquire information stated explicitly in required text material. Many definitions, facts, concepts, and other content understandings identified as essential call for no more than literal understanding on the part of our students. The suggestions in this chapter should prove useful in this case. However, literally stated information is also critical to objectives which demand higher thought processes. Therefore, it is important for the teacher to

realize that successful literal reading is critical to success at the next levels as well. To the degree that mathematical reading is a hierarchical process, a critical point to be made here is that these guiding activities will provide the very foundation for success at higher levels of mathematical reading.

- **Summary**

Success at the literal level of mathematical reading requires that the student know why he is reading a particular assignment, and what he hopes to find there. To this end the teacher can provide guidance in the form of specific purposes for reading, or by providing study guides which focus the student on the content understanding to be gained. The study guide offers the student additional opportunities to speculate, locate, verify, and correct by referring to the assigned text material. The next chapter contains suggestions for continued guidance toward reading success when mathematical objectives require the student to identify important relationships among several literal meanings.



Chapter 6

ANALYZING RELATIONSHIPS

Given a specific assignment, chapter, or unit in mathematics, if your students have mastered the pronunciations and meanings of the essential symbols and have drawn from the assigned reading those literally stated facts deemed essential to the content understandings, they are ready to proceed to the next level of mathematical reading—analyzing their relationships in the larger context of mathematics. Success at this level requires the student to identify and explore critical relationships among words or ideas. For example, given a group of symbols, the student must identify the topic they describe, adding other essential symbols and discarding irrelevant ones. Or, given several factual details from a text, the student is forced to decide what the author really means by drawing a conclusion from the statements. Or, having gathered facts presented in a word problem, he must discard the irrelevant and arrange the rest in an order which represents a true mathematical statement, such as equality. This chapter contains several suggestions for guidance at this level of the reading process.

• Structured Overview

The structured overview is a technique that offers teachers help in performing their content analysis (as described in Chapter 1) as well as possibilities for student guidance at the relationships level of reading. The directions for constructing and using a structured overview are as follows:

1. Analyze the vocabulary of the learning task and list all words that you feel are representative of the major concepts that you want the students to understand.
2. Arrange the list of words until you have a diagram which shows the interrelationships among the concepts particular to the learning task.
3. Add to the diagram vocabulary concepts which you believe are already understood by the students in order to depict relationships between the learning task and the discipline as a whole.

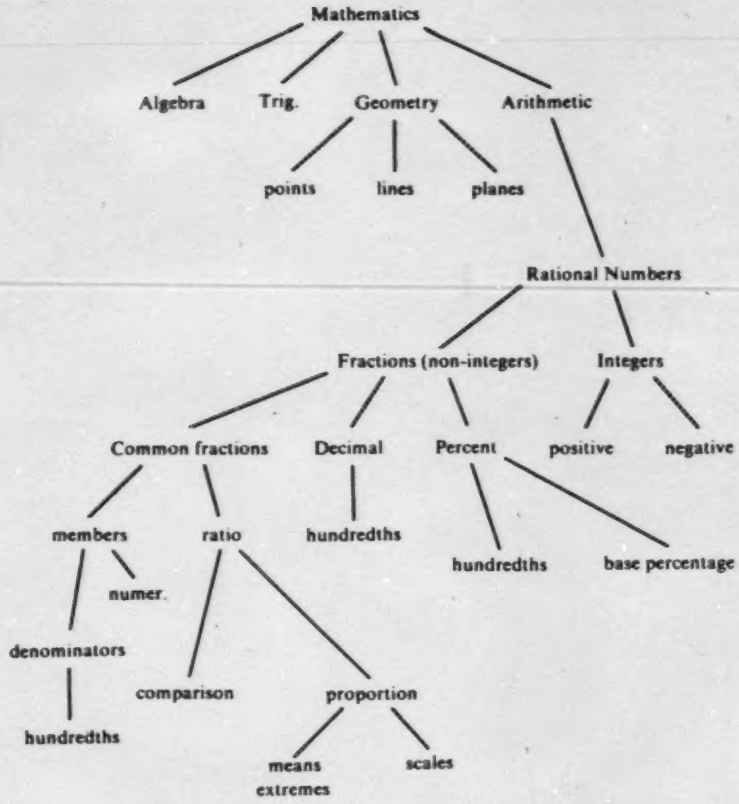
4. Evaluate the overview. Have you depicted major relationships clearly? Can the overview be simplified and still effectively communicate the relationships you consider to be most important?
5. When you introduce the learning task, display the diagram to the students and explain briefly why you arranged the words as you did. Encourage the students to supply as much information as possible.
6. During the course of the learning task, relate the new information to the structured overview as it seems appropriate (Earle and Barron, 1973, p. 84).

Examine carefully the first four steps as listed above. Note that they involve selection of vocabulary essential to the learning task, one of the key steps in the content analysis (Chapter 1). They also direct the teacher to arrange the words or other symbols until a schema is produced which shows the interrelationships among the terms. Teachers have found this approach to be very useful in revealing the content objectives. For practice in manipulating and arranging terms to construct a structured overview, Appendix C presents a list of terms selected by one teacher as essential to a brief unit on averages. A good way to get the *feel* of constructing a structured overview is for the teacher to separate the terms by cutting along the dotted lines. This will allow the exploration of several arrangements with ease. Even better, select a list of essential terms from an upcoming unit of your own content and try to make an arrangement of terms according to the above directions. A structured overview can be constructed by one teacher alone; however, it is often productive to invite a colleague or two to join in, since several points of view add perspective to the resulting schema.

Examples 1–6 are structured overviews constructed by teachers, following the above directions.

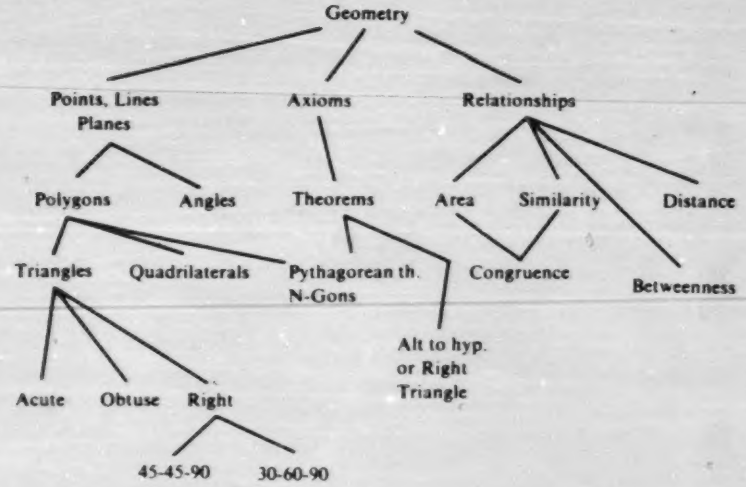
Teachers who have constructed structured overviews have reported that the process has helped them clarify their content objectives, since they are required to rethink with care the conceptual relationships among the several terms they regard as essential to a given unit of study. Since single terms are used to construct the overview, the resulting schema is rarely self-explanatory. Rather, the particular arrangement of terms chosen by the teacher must be explained so that relationships among terms are clarified. This characteristic offers possibilities for teaching students these important relationships, as will be noted by reference to steps five and six of the directions above. It is here that the teacher encourages the students to examine the relationships, eliciting information and presenting his own explanations.

Example I



Employed in the mathematics classroom according to previous directions, the structured overview helps students to see the "big picture" of mathematics study, noting relationships among key terms, as well as the relationship of present, past, and future topics of study. Many teachers have found that the principle of the structured overview offers considerable opportunity for active student involvement as well. For example, after selecting the terms as per the directions, write or print each term on a slip of paper or small card (or have the students do it). Provide one set of these cards for each group of four or five students in class. Ask the students to explore relationships among the terms: "Which of these terms represents an idea that includes all the others?" "Arrange these terms

Example 2



Example 3

Factoring-Changing Form—from Addition to Multiplication

Arithmetic—numbers

↓

Algebra

Polynomials—classified by degree or number of terms

↓

Monomial
Binomial
Trinomial

Distributive Property for linear equations
Difference of Squares
Perfect Squares
Completing the Squares
Trinomials

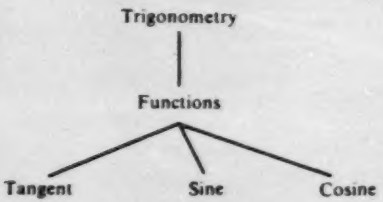
} Quadratic Equations
Simplifying Fractions

Intermediate Algebra

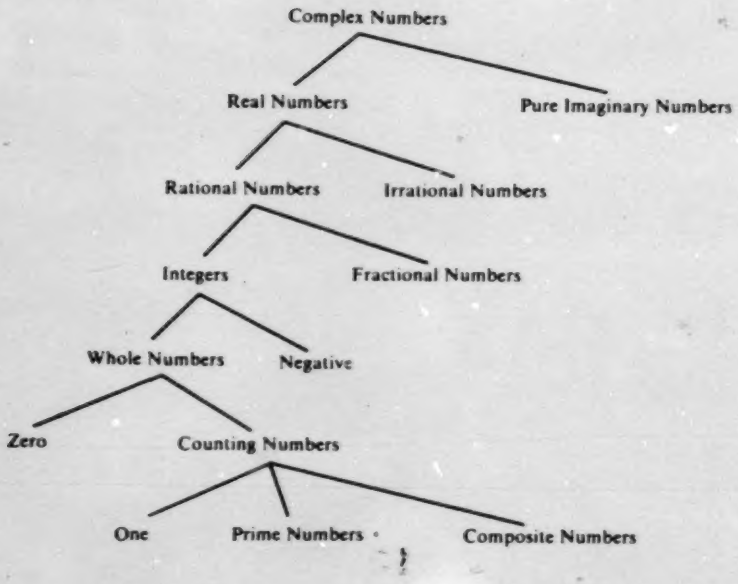
Geometry

in order of size (or importance)." "Arrange these terms so that they represent a sequence of steps in the mathematical operation we have been studying." "Arrange these terms in any sensible way that you can explain." Fewer terms presented at any given time usually represent a task which is relatively easier to deal with; more terms represent a more complicated problem. Some teachers use the concept of the structured overview at examination time, allowing students to work together to design a sensible arrangement of terms, then requiring an individual explanation, either oral or written. The concept of the structured overview deserves exploration. Its possibilities are limited only by the individual teacher's imagination.

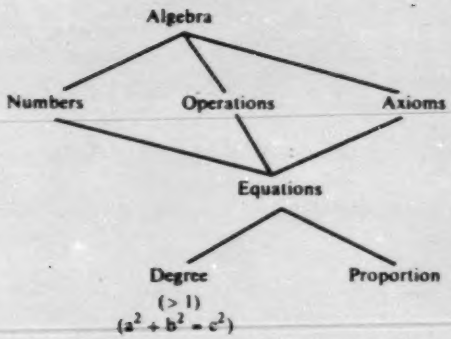
Example 4



Example 5



Example 6



- **Manipulating Symbol Relationships**

In addition to the structured overview, there are any number of exercises which can be designed to strengthen the student's grasp of important conceptual relationships among the vocabulary essential to a given unit of study. The following examples are but a few types.

Example 1

Directions: Select the idea in each group which includes all the other words in the group. Look at this example.

snow
Christmas
winter
cold

The correct choice here could be winter because the idea *winter* includes all the others. You must be able to explain your choices.

1. Composite Numbers
   One
   Whole Numbers
   Prime Numbers
   Zero
2. Closure
   Distributive Law
   Multiplication
   Associative Property
3. Whole
   Numeration
   Integers
   Rational

Example 2

Directions: Select the word that does *not* belong in each group. Look at this example.

hammer
rake
shovel
hoe

The correct choice could be *hammer* since all the others are garden tools and a hammer is a carpenter's tool. Be prepared to prove your choices.

1. GCF
   Prime Factors
   Common Factors
   Greatest Common Divisor
2. Prime Numbers
   Composite Numbers
   Multiples
   One
3. Prime Numbers
   Multiples
   Prime Factors
   LCM

Example 3

Directions: Each group of 3 to 5 students receives one set (three cards). *Scoring* 5^0—Identify topic they relate to 5^1—Explain relationship (2 terms) 5^2—Explain relationship (3 terms)	PROPORTION 2
MULTIPLICATIVE INVERSE 3	
FACTORS 1	RECIPROCAL 3
MULTIPLES 1	COMPLEX FRACTION 3
DIVISIBILITY 1	13 7/8 4
RATIO 2	4.5 4
MIXED FORMS 2	π 4

Example 4

Directions: Draw a circle around the larger measurement in each pair listed below. You must be able to *prove* your choices.

1. centimeter	decimeter	6. kilogram	ounce
2. gram	milligram	7. megagram	microgram
3. meter	kilometer	8. gram	ounce
4. mile	meter	9. decigram	milligram
5. metric ton	English ton	10. inch	centimeter

Example 5

Directions: There are five words on each line below. Circle two words on each line which have something in common such that when you have done so, the remaining three will also have something in common, other than not being selected.

Is there more than one pair which you could select on each line that would satisfy these requirements?

1. inch	pint	yard	gallon	mile
2. liter	foot	gram	rod	quart
3. ounce	gallon	centiliter	cup	ton
4. milligram	gram	megagram	decigram	microgram

Example 6

Directions: The following words are all considered to be key words that signal the need for performing one or more operations in mathematics. The four operations form the categories into which each word fits. Place each word under the operation which it suggests.

total	difference	divided by	how many times more
sum	add	left over	product
minus	times	subtract	in all
how many would		altogether	plus
how many times			

Addition	*Subtraction*	*Multiplication*	*Division*

Example 7

Directions: In each set of words given below, you will find the following to be true: the first two words of the set have a definite relationship to each other (like latitude: parallels). Following these two words is a single word followed by a blank. Below the blank are three words. You are to choose the word that is related to the word preceding the blank in the same way as the first two words are related. Underline your choice.

Example: editor: newspaper :: producer: _____

program, *play*, building

(An editor is responsible for the publishing of a newspaper; a producer is responsible for the production of a play.)

1. polygon: circle :: polyhedron: _____

cube, square, sphere, box

2. diameter: circle :: base: _____

square, cube, quadrilateral, triangle

3. area: square units :: volume: _____

inches, cube units, weight, pounds

4. triangle: altitude :: quadrilateral: _____

height, base, length, width

Example 8

Angle Relationships: (By p. 77 in Schacht & McLennan)

Directions: Place an "X" on the blank to the left of each word in the third column which could form a meaningful relationship when used with the other two words in the section.

1. acute	adjacent	____complementary
		____supplementary
		____vertical
		____equal
2. obtuse	adjacent	____complementary
		____supplementary
		____equal
		____corresponding
3. interior	supplementary	____exterior
		____adjacent
		____right
		____triangle
		____alternate
4. vertical	central	____equal
		____obtuse
		____acute
		____supplementary
		____complementary

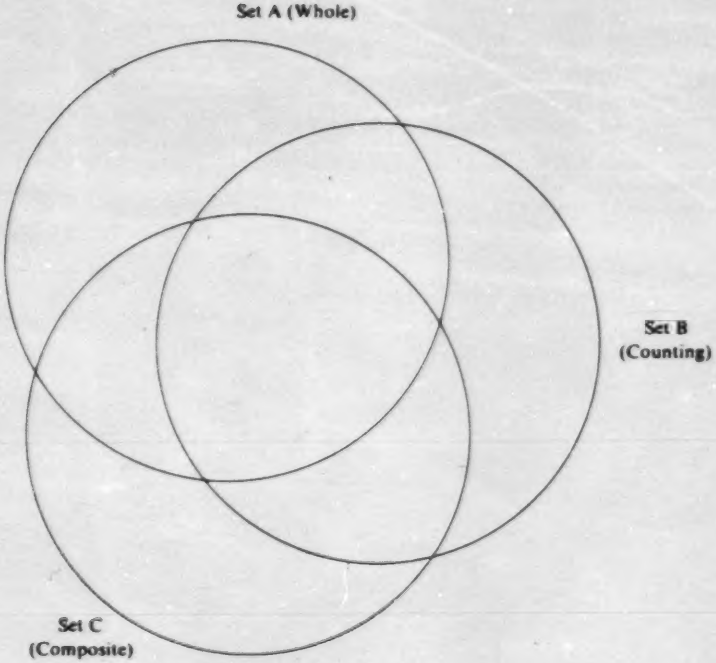
5. corresponding complementary ____acute
____obtuse
____equal

Source: H. L. Herber, *Teaching Reading in the Content Areas.* Prentice-Hall, 1970, p. 174. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Example 9

Directions: Sets A, B, and C below represent the sets of whole, counting, and composite numbers. Write each of the following terms in the set where it belongs. Be careful! Some terms may not belong at all.

number	twenty	cardinal
one	composite	prime
zero	even	odd
fractions	natural	large
twenty-one	two	seven
placeholder		factor



Example 10

Directions: Each team of three to five students receives two or three sets of cards. Your task is to "build" equations by starting with the basic equality in each set and following the directions through card 2 and card 3. When you finish, write the equations you have built on a sheet of paper and give them to another team to solve.

$^1X = 3$	^{2}Add 2 to both members	^{3}Multiply both members by -1
$^1X = -2$	^{2}Add 2 X to both members	^{3}Divide both members by 1/3
$1X = 7$	^{2}Divide both members by 1/2	^{3}Add $-2X$ to both members

Example 11

Directions: Check the ordered pairs that have the same relationship as the first two pairs in each set.

(0, 1) (1, 2) ____ (11, 12)
____ (8, 7)
____ (2, 2)

(1, 1) (2, 4) ____ (2, 2)
____ (3, 9)
____ (7, 49)

(8, 2) (10, 4) ____ (11, 5)
____ (11, 12)
____ (6, 0)

Example 12

Directions: Supply the missing numbers that will continue the relationships.

(12, 4), (18, 6), (21, ____), (6, ____)
(3, 9), (5, 25), (11, ____), (0, ____)
(2, 9), (5, 12), (8, ____), (11, ____)

The reader of this volume will note that all of the activities listed are designed to encourage the student to explore and manipulate conceptual relationships. Needless to say, it is virtually impossible to identify relationships among literally stated ideas or definitions unless those literal meanings have been first grasped by the student. Hence, another bit of support concerning the hierarchical nature of our definition of mathematical reading.

• Identifying Relationships in Context

The above activities tend to involve clusters of essential mathematical terms, with the student strengthening his grasp of the conceptual relationship through engaging in the particular activity. Sometimes, however, a particular content objective calls for the identification of meanings unstated; that is, implicit in textual material or some other mathematical context. In this case, the teacher can design guiding activities which focus the student on the essential literal meanings, then extend the guidance to include a focus on certain possible relationships that should be considered by the students. The following activities are examples of this type of guidance. Note in particular the consistent focus on proving or supporting statements of relationship, based usually on meanings presented literally in text or lecture presentations.

Example 1

Directions: Using your answers from page M-2A and your multiplication chart, check (✓) those statements that are true. You must prove your answers.

_____ Any number times one equals that number.
(Hint: page M-2A, problems f, g, l, m, 2)

_____ $3 \times 7 = 2 \times 7$
(Hint: page M-2A, problems A, K)

_____ $8 \times 3 = 3 \times 8$
(Hint: page M-2A, problems J, Q)

_____ $8 \times 9 = 8 \times 7$
(Hint: page M-2A, problems O, V)

_____ Zero times any number equals zero
(Hint: page M-2A, problems R, V, Y)

_____ We can change the order of the number of a multiplication problem and not change the answer
(Hint: page M-2A, problems A, K, J, Q, R, E, O, T)

Example 2

Part I (Literal)

Directions: Check the statements below which tell what the author *said* in the text (page 105, section A).

_____ 1. The perimeter of a polygon is defined as the sum of the length of its sides.

_____ 2. The distance around a circle is its perimeter.

_____ 3. Perimeter is measured by the number of units it contains.

_____ 4. You can find the perimeter of any polygon by adding the lengths of the side directly.

Part II (Relationships)

Directions: Check the statements below which tell what the author *means*. Your responses to Part I should help you.

_____1. Since a circle is not a polygon the distance around it is its circumference.

_____2. If you know a figure has all equal sides you can find the perimeter by multiplying instead of adding.

_____3. 56 sq. in. is an example of a measurement of perimeter

_____4. $P_{\Delta} = a + b + c$

Example 3

Directions: If *a* represents the first number and *b* the second number, write the following in algebraic symbols.

_____The sum of two numbers

_____The difference of two numbers

_____The product of two numbers

_____The square of the first number

_____The square of the second number

_____The difference of the square of the two numbers

_____The square of the first, plus twice the product of the two numbers, plus the square of the second number.

Example 4

Directions: Complete each of the following statements of relationship. You are free to consult with your work partner. It will also be useful to consult your text (Chapter 3), although you will *not* find the actual answers, only information that you can use to find the answers.

1. The _____ of a _____ is three times that of a cone having equal radii and equal heights.

2. The _____ _____ of a cylinder is twice the area of one base plus the lateral area.

3. The difference between a triangle and a tetrahedron is _____ _____.

4. The similarity between a sphere and a circle is _____ _____.

5. Length, area, and volume are all _____ _____.

Example 5

Directions: Referring to the graph on page 324, try to prove each of these statements. If a statement is true, mark it so. If it is not, change the underlined part to make the statement *always true*.

_____ 1. The x-coordinate of the point (3, −2) is 3.

_____ 2. The y-coordinate of any point in quadrant II is negative.

_____ 3. The y-intercept of the graph of $5x - 3y = 15$ is 3.

_____ 4. The slope of the graph of an equation written in the form $y = mx + b$ is m.

_____ 5. The x-intercept of the equation $y = 3x = 5$ is −5.

_____ 6. If a system of equations has one and only one root, then the equations are called inconsistent equations.

_____ 7. The substitution method (axiom) may be used to solve a system of linear equations.

_____ 8. The graph of $\begin{matrix} x & 0 \\ y & \end{matrix}$ is quadrant III.

_____ 9. If a line AB forms two half-planes, then line AB is in neither half-plane.

_____ 10. A vertical line through A: (2, 3) contains the point B: (2, 1).

Example 6

A. Subtraction is the inverse of addition

Division is the inverse of _____

Taking a root is the inverse of _____

$4 = 2^2$ is the same statement as $\sqrt{4} = 2$

$M = Q^2$ is the same statement as $\sqrt{M} = Q$

$25 = 5^2$ is the same statement as _____

B. Express the following phrases in symbols. The first one is done for you.

1) *The sum of two square roots* $\sqrt{A} + \sqrt{B}$	*the square root of a sum* $\sqrt{A + B}$
2) *The difference of two square roots*	*the square root of a differences.*
3) *The product of two square roots*	*the square root of a product*
4) *The quotient of two square roots*	*the square root of a quotient.*

C. Fill in the blanks in order to solve the equations. The first one is done as an example.

1) $\sqrt{16} + \sqrt{9} \stackrel{?}{=} \sqrt{16 + 9}$

$\underline{4} + \underline{3} \stackrel{?}{=} \sqrt{25}$

$\underline{\quad} \neq \underline{\quad}$

2) $\sqrt{100} - \sqrt{64} \stackrel{?}{=} \sqrt{100 - 64}$

$\underline{\quad} - \underline{\quad} \stackrel{?}{=} \sqrt{36}$

$\underline{\quad\quad\quad} \stackrel{?}{=} \underline{\quad}$

3) $\sqrt{9} + \sqrt{16} \stackrel{?}{=} \sqrt{(9)(16)}$

$(\quad)(\quad) \stackrel{?}{=} \sqrt{\quad}$

$\quad \stackrel{?}{=} \quad$

4) $\frac{\sqrt{100}}{\sqrt{25}} \stackrel{?}{=} \sqrt{\frac{100}{25}}$

$\frac{(\quad)}{(\quad)} \stackrel{?}{=} \sqrt{4}$

$\quad \stackrel{?}{=} \quad$

Compare your responses to Part B with your responses to Part C. Then fill in the blank space in each statement below with "does" or "does not." Try to get all members of the group to agree before checking with the teacher.

1) The sum of the square roots _____ equal the square root of the sum.
2) The difference of the square roots _____ equal the square root of the difference.
3) The product of the square roots _____ equal the square root of the product.
4) The quotient of the square roots _____ equal the square root of the quotient.

Example 7

Directions: One very difficult thing to do in word problems is to separate the essential information from the *unnecessary* information. For each of the problems on page 86 in your text, decide whether the information listed below is essential (E) or Unnecessary (U) in solving the problem.

1) Mrs. Jackson_____ each month_____
16 gallons_____ year_____
milk_____

2) Pat_____ already_____
saving_____ $8.47_____
ski jacket_____ more_____
$14.95_____ 75¢ per hour_____
babysitting_____

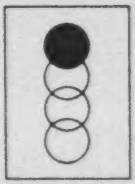
- **Role of the Content Analysis**

Content determines process. If the teacher's content objectives require only meanings literally presented in text or by lecture, teaching ideas similar to those presented in earlier chapters will suffice. Many mathematical tasks, however, require the student to analyze relationships if he is to be successful. In this event, guiding activities suggested in this chapter should be useful in helping the student master both the mathematical content and the necessary reading-

learning process. The activities described in this chapter are *not* necessary if 1) the desired content understandings do not require this relationships level of reading or 2) the students have no difficulty at this level of reading, given the particular topic or material at hand.

• Summary

This chapter suggests activities which are designed to guide students to an understanding of conceptual relationships by manipulating literal definitions or other ideas and exploring their relationships to grasp meanings that are implied, rather than explicit. It is important for the teacher to recognize that this type of activity is useful only when it guides the students to consider the relationships identified as essential to the content objectives. Otherwise, although they can be useful and even fun, the activity detracts from the mathematical learning task at hand. It is important to let the content determine the process, not the inverse. Guidance at this level, when required by the objectives, provides the foundation for the successful solution of word problems, the topic treated in the following chapter.



Chapter 7

SOLVING WORD PROBLEMS

The solution of word problems is one of the most sophisticated of all tasks in mathematics, at least from the reading point of view. It is here that the reader must perform accurately all the previous levels of reading—perceiving symbols, attaching literal meaning, and analyzing relationships—and extend that knowledge toward the correct or creative solution of a mathematical problem, actual or simulated. It is in problem solution that the hierarchical nature of our reading definition is perhaps most apparent, for each specific problem situation requires the student to proceed carefully through a sequence of steps, including those levels described earlier in the volume, eventually arriving at an algorithm which represents a clear mathematical statement of the entire problem situation. If these requisite steps are completed correctly, given the problem in question, all that may remain is some rather low level computation, not treated in this volume, but judged by many mathematics teachers to be the easiest part of problem solution. Any other approach is likely to result in disaster. This chapter offers suggestions for guiding your students in solving word problems in a rational way, when the content objectives so indicate.

• General Approach to Word Problems

With few variations, experts in both reading and mathematics agree that the following steps represent a sound general approach to the solution of word problems.

Step 1: Read through the problem quickly. Try to obtain a general grasp of the problem situation and *visualize* the problem as a whole. Don't be concerned with the actual names, numerals, or values.

Step 2: Examine the problem again. Try to understand exactly *what you are asked to find.* This may be stated as a question or command. Although it often comes at the end of the problem, it may appear anywhere in the problem.

Step 3: Read the problem again to note what *information* is *given.* At this point you are looking for exact numbers and values.

Step 4: Analyze the problem carefully to note the *relationship* of information given to what you are asked to find (Steps 2 and 3). Note information which seems to be missing. Also surplus information.

Step 5: *Translate* the relationships to *mathematical terms*. Indicate both the values and operations. This almost always involves planning a *sequence* of steps which correspond to the operations. The end result will be one or more mathematical sentences or *equations*.

Step 6: Perform the necessary *computation*.

Step 7: Examine the solution carefully. *Label* it to correspond to what the problem asks you to find. Finally, check the value against your grasp of the problem situation to judge whether it seems *sensible*.

The above general approach to solving word problems corresponds closely to the model of mathematical reading used throughout this volume. Note that all the steps require the accurate perception of symbols—general, technical, and special. Steps 1, 2, and 3 require the reader to attach literal meaning. Steps 4 and 5 require him to analyze relationships among the explicitly stated details. Steps 6 and 7 demand that he apply his computational ability to the set of relationships and judge the result critically in light of the original purpose.

- **Role of the Content Analysis**

At this point it may seem redundant to mention the role of the content analysis in guiding students through the reading process. However, it is at the problem solution level that we often tend to violate most flagrantly the tenets presented and reinforced throughout this volume. For example, some teachers who spend considerable time and energy guiding students through the lower levels required by the study of a given topic forget that each word problem represents a synthesis of *all* levels of the reading-thinking process. Having guided students carefully in grasping the definition of essential symbols, it is tempting to move directly to word problems in that topic, without realizing that each problem itself requires the *entire sequence*. When students experience difficulty, the teacher may express sympathy (or anger) but often assigns a few more problems of the same type (which the students cannot do either). This attempt to treat difficulties in the reading process by prescribing generous doses of additional content usually is not productive.

- **Tried but Not True**

What can the teacher do? Some teachers refer the student to the general approach described above, often reproducing the steps on a

large chart for prominent display in the classroom. Since mathematical content and the related reading process are intimately connected, however, it is most difficult for many students to transfer a set of general directions for problem solution to the particular problem at hand. In short, when the process is described in one place (on the wall of the classroom, for example) and the content problem in another (page 251 in the text), it takes a very independent student to achieve the rather sophisticated integration required to apply one to the other successfully. This is not to say that emphasizing general approaches to the solution of word problems is not sometimes useful. Such emphasis is beneficial, especially for more able students. However, in the event that it does not provide the necessary degree of guidance for the others, further guidance is necessary.

Another approach mathematics teachers often use is the careful teacher-led attack on a model problem by the whole class. Using a typical problem dealing with a certain topic of study, the teacher demonstrates a logical and sequential attack on the problem, eliciting information from the class throughout. Once again, some students gain insight. Unfortunately, after such a demonstration, the teacher often concludes that *he* was doing most of the problem solution, with a few students being helpful and the majority trying their best to pay attention in a passive manner. The benefits remain minimal.

Sometimes teachers suggest that students take a rather superficial approach to the task of solving word problems. These superficial approaches include such mechanical devices as "The last sentence in the problem always tells what is asked for." or "When you see 'signal words' like *total, sum,* or *product,* you simply perform the operation according to the cues given by these signals." Unfortunately, this approach leaves much to be desired if students are expected to develop a rational approach to problem solution. With the least variation in problem wording, the student becomes hopelessly confused. Moreover, in problems requiring more than one simple operation, signal words alone cannot be translated into a reliable sequence. In fact, there is no simple way to solve word problems. *Instead, students must be guided in the appropriate sequence through a variety of problem situations.* Guidance is gradually reduced, as student competence (and confidence) increases, until it is withdrawn altogether when the student is able to solve a particular type of problem independently.

- **Firm Foundations**

One of the best assurances that students will be successful in their approach to word problems is the judicious use by the teacher

of the suggestions presented in earlier chapters. The prognosis for problem solution is improved considerably in a particular topic of study if the teacher does not assume, but is certain, that students can perceive the essential symbols quickly and accurately, have a firm grasp of the definitions and other explicitly stated key ideas, and have spent adequate time in exploring and manipulating many possible relationships among those terms or ideas. Judicious use of the suggestions presented earlier in this volume, therefore, is essential to success at this level of reading. (If, for example, the reader has obtained this volume and has chosen to begin with this chapter, it is strongly recommended that he return to Chapter 1.) To the degree that the model of reading we are using is applicable to mathematical reading, successful reading at the three previous levels is requisite to success in solving word problems.

- **Symbol Perception**

Given a firm foundation, most students will have mastered the correct pronunciations of terms essential to the topic or operation under study before word problems are undertaken. (Chapter 3 provides numerous suggestions for teacher guidance at this level.) Although symbols selected for word perception emphasis will not of course include all the terms appearing in a given word problem, the point is that the *essential* terms should provide no obstacle to the student. In the event that there remain some students who cannot perceive the symbols accurately, there are two major approaches the teacher may take. First, during the content analysis the teacher selects those key symbols from the problems assigned which may present perception obstacles to the students. When the assignment is given, he will spend from one to five minutes making sure that all students know how to pronounce these terms. Second, the teacher can encourage students to ask another person—teacher or student—how to pronounce a troublesome symbol that appears in an assigned problem. A particular student with severe perception problems may be allowed to solve problems with another student whose perception is more accurate (but whose ability at higher levels is similar).

- **Guided Solution**

Since most experts agree that a problem approach similar to that stated earlier is generally effective, necessary teacher guidance consists of combining the general approach with the content of a specific problem to provide the student with a sequential framework leading to solution. For example, a common algebra problem is stated as follows:

> In a tank there are 100 pounds of a solution of acid and water which is 20% acid. How much water must be evaporated to produce a solution which is 50% acid?

One teacher formulated the following set of questions for the students to use as a guide. Notice how the general sequence of questions has been made specific to the problem under attack.

Example 1

a. Since only the water evaporates, will the amount of acid stay the same?

b. What are you asked to find? Express it mathematically.

c. How much acid do you have in the beginning? Express it mathematically.

d. How much solution do you have after the proper amount of water has evaporated? Express it mathematically.

e. How much acid do you have after the water was evaporated? Express it mathematically.

f. What equation can you write to show the relationship between the original amount of acid and the final amount?

g. Solve the equation. Label your answer. Does it seem sensible?

The above sequence of questions has two very important characteristics. First, teacher guidance is added by a sequence of specific questions that *simulate* the process of reading word problems. Second, each student is responsible for actively considering each step of the process. Additional amounts of guidance can be provided by simply stating the questions in a different form, one which presents the actual components of the problem in an appropriate sequence for examination and manipulation by the student, as follows:

Example 2

a. True or False? If a choice is false, correct it.

_____ The original solution is 50% acid.

_____ Acid does not evaporate.

_____ Water does not evaporate.

_____ The final solution will be stronger.

b. You are asked to find

_____ x

_____ the amount of water to be added

_____ the pounds of solution

_____ the number of pounds of evaporated acid

c. The original amount of acid is

_____ 50% of 100

_____ 20 lbs.

_____ .20 × 100

_____ .50 (100 − X)

d. After the proper amount of water has evaporated, the amount of solution can be represented by

_____ x
_____ 100 − x
_____ 20 + x
_____ .20x + 100

e. The final amount of acid is

_____ 50% of 100
_____ 20 lbs.
_____ .20 × 100
_____ .50(100 − x)

f. Which mathematical expression of this problem is true?

_____ .5x = 100 − 20
_____ 20 = 20
_____ .50(100 − x) = .20 × 100
_____ .20x = .50(100 − y)

g. Solve the problem in this space.

The point here, of course, is not that the teacher of mathematics should attempt to structure every assigned problem in the above manner. However, this type of guidance can be very useful as an instructional tool—using real word problems in such a way that students are immediately aware of the hierarchical and sequential steps which make up a successful reading. Following are several other examples of problems at different levels which were structured by teachers of mathematics to guide their students to a correct solution, while at the same time emphasizing and clarifying the reading process involved.

Example 3

Mary bought 2 yds. of ribbon at 20 cents a yd. and some cloth for 50 cents. How much change did she get from a dollar bill?

A. True or False? If an answer is false, cross out the wrong word or words and correct them.

_____ Mary bought 2 yds. of ribbon.
_____ The ribbon cost 15 cents a yd.
_____ The cost of 2 yds. of ribbon was 50 cents.
_____ She bought 2 yds. of cloth at 50 cents a yd.
_____ The price of the cloth was 50 cents.
_____ She paid the clerk with a five dollar bill.

B. Underline the question in the problem.

C. Check (✓) the correct answer below.

In solving this problem you are asked to find

_____ How much change she received.

_____ The price of ribbon and cloth.

D. Number the steps below in order

_____ Label the answer.

_____ Subtract the total amount from $1.00.

_____ Find how much the ribbon cost.

_____ Find the total amount Mary spent.

E. Use the space below to solve the problem, using the steps from D (above).

Example 4

(Dolciani, M.P., et al. *Modern School Mathematics, Algebra 1.* Boston: Houghton Mifflin, 1967, 125.)

An airmail parcel may weigh from 8 ounces up to 70 pounds, but the sum of the length and girth may be no more than 100 inches. What are the dimensions (length, width, height) of the longest acceptable air mail parcel, in the shape of a rectangular solid, that is twice as long as it is wide and that is two inches higher than it is wide?

*What are you asked to find?

_____ height _____ cost _____ width _____ dimensions

_____ postage _____ girth _____ weight _____ length

*What mathematical expressions are useful in this problem? Write each word you selected (above) beside the appropriate expression.

_____ x _____ $2x$ _____ $3x$ _____ $4x$

_____ $x + 1$ _____ $x + 2$ _____ $x + 3$ _____ $x + 4$

_____ $x - 1$ _____ $x - 2$ _____ 100 _____ 70

*What mathematical relationships are useful in this problem? Check the statements that are true.

_____ girth means width and height

_____ total dimensions are not more than 100

_____ an equation is a statement of relationship

_____ size is related to weight

_____ length, width, and height must equal 100

*Based on all the above questions, which of the following is correct? You *must* be able to explain your choice.

_____ $x + 2x + 3x = 70$

_____ $3x - 1 > 78$

_____ $x + 2x + (x + 2) \leq 100$

_____ $3x + 3 = 100$

*Solve the equation you chose. What dimension does this give you?

*Use your solution to attach values to the first question.

*Check your answer.

Example 5

Jack had 50 cents. He bought 4 post cards at 5 cents each. How many post-cards at 10 cents each could he buy with the money he had left?

A. Check one answer.

In this problem you are trying to find

____the number of 10 cents cards Jack can buy

____the price of the post cards Jack bought

____the money Jack had to begin with

B. As you read the problem, write the correct amount from Column B before each space in Column A

Column A	*Column B*
_____The amount of money Jack had to start with	30 cents
_____The amount he paid for 4 cards at 5 cents each	50 cents
_____The amount he had left after he bought 4 cards at 5 cents each	20 cents

C. With your work so far, you should be able to solve the problem now. Be sure to label your answer.

Example 6

A living room is 16 ft. by 20 ft. The rug, also rectangular, measures 12 ft. by 15 ft. How many square feet of the floor remains uncovered by the rug?

In the space below, draw a rectangle to represent the living room.

Label the length and width.

Draw in the rug. Label the length and width of the rug.

What does the problem ask you to find? Shade this part in your drawing.

What relationship are you dealing with in this problem?

_____Floor area plus rug area _____Floor area minus rug area

_____None of these _____Floor area times rug area

Solve the problem and label your answer.

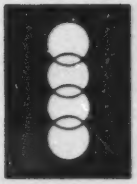
In all the above examples, the teacher has seen the process of problem solving as being composed of several of the levels of reading mentioned throughout this volume. In each case, guidance has been provided in various amounts which simulates the teacher's own attack on the specific problem through the sequencing of questions which lead to the correct solution. Once the goal is understood, the appropriate sequence of steps is revealed by the teacher's careful

examination of the specific problem at hand, with variations depending on the teacher's own view of the particular process. Once the sequence is established, moreover, guidance can be adjusted to student needs in an almost infinite number of ways. Although there is no formula to ensure the correct amount of guidance, the teacher's basic purpose is to bridge the gap between his own perception of the solution process and the student's. When that gap is formidable, the necessary steps are presented in great detail, with no step left out, however small. In addition, alternatives provided by the teacher can provide a framework within which to attack the problem. When the gap is narrower, the teacher can focus the student on only certain steps in the solution sequence, perhaps relying on questions or suggestions rather than alternatives to manipulate.

The important point to remember is that the content of a particular problem is the focus of the activity, with the solution as the content objective. The necessary reading process, although it is the individual teacher's perception, probably corresponds closely to the model we have been using throughout this volume: Word Perception, Attaching Literal Meaning, Analyzing Relationships, leading in sequence to Problem Solution.

• Summary

This chapter has suggested guiding activities which represent a synthesis of all information presented in earlier chapters. The purpose has been to suggest means of guiding students to success in their approach to word problems in mathematics. Beginning with the content analysis of a particular problem, the teacher can formulate guiding activities at each of the necessary levels of the reading process, according to the needs of the individuals and groups within a given class. Although guidance at this level of the process can and should include many of the suggestions presented in earlier chapters, the essential point stressed here is the arrangement of appropriate activities in a coherent sequence, all of which will lead the students through the process to the correct problem solution. This sort of guidance helps the student to be successful in a rational fashion, rather than experiencing trial and error success. It places the burden of responsibility on the student, ensuring his active involvement. However, it also assumes teacher involvement in planning the sequence of activities, based on his original goal (content analysis) and his understanding of the problem solving approach (reading-thinking process).



Chapter 8

VOLUNTARY MATHEMATICAL READING

There is a final important mathematical reading skill that has not been mentioned thus far—voluntary reading of a recreational or supplementary nature. Thus far this volume has offered suggestions for guiding students in perceiving symbols, attaching literal meaning, identifying relationships, and solving word problems. These, however, are all skill areas involving cognition or comprehension. Voluntary reading in mathematics is reading activity engaged in by students because they *want* to, not because they *have* to. And this depends largely on their attitude or motivational level regarding the subject, the teacher, and the mathematics classroom. Needless to say, many factors influence the attitudes or motivations of students in mathematics classes. Many of these factors (e.g., experiential or emotional in nature) are those over which we as teachers have very little control. In all probability, many factors which influence attitude or motivation are not yet known. Nonetheless, of the many variable factors which help to shape student feelings about mathematical reading (and, indeed, mathematics in general), the two that we as teachers can control most directly are perceived *student success* and availability of *appealing materials*. This chapter explains briefly how teachers can promote these two ideas in the mathematics classroom.

• Student Success

Student success can be defined as the degree to which the student feels he is mastering required reading and learning in mathematics. Note that this success must be perceived by the student himself, not merely by the teacher. One of the most common of all maxims is that we like to do what we do well; i.e., that which we feel successful in. On the other hand, given the choice, we tend not to do that which we are not good at. This is an example of the simplest connection between affect, or feelings, and cognition, or learning. Successful learning makes us feel good about the subject; feeling good about the subject (and ourselves, therefore) makes us eager to learn more and makes that learning easier, since we will be more apt to engage in that activity voluntarily.

It will be noted that the first seven chapters of this volume have

been designed to show the teacher of mathematics how to help ensure reading success for all students. Based always on the mathematical objectives as judged by the teacher, as well as on the varying abilities of the students, the foregoing suggestions for guiding students through the reading-learning process in mathematics should do much toward helping students achieve success in mathematical reading. Teachers should take care that this success is made very clear to the individual student, in cases where he does not himself perceive this success. Guidance through the reading process as described to this point should contribute substantially to better feelings about both self and mathematics, and should pave the way for voluntary reading of interesting recreational and supplementary mathematical material, when they are available.

- **Appealing Materials**

The second factor which can help encourage voluntary mathematical reading is the availability of appealing materials; that is, immediate access to interesting supplementary and recreational material written on a variety of difficulty levels. Any marketing expert knows that appealing materials, attractively displayed, will "sell" the customer. Following this rule, the teacher can seek out material of high appeal; display it prominently and attractively in the classroom; and provide time throughout the week for students to browse, select, and read on a purely voluntary basis. In almost every classroom there are times appropriate for free reading on an individual, group, or whole class basis. In fact, some teachers make a point of scheduling such time on a regular basis at least once each week during regular class time. Although individual class schedules vary, ten to thirty minutes can always be found if the teacher's priority demands it.

With regard to materials display, the key word is *attractiveness*. Books, games, magazines, and kits can be arranged on built-in bookshelves, uncluttered tables designated for that purpose, tops of heating or cooling units, chalkboard trays, inexpensive racks built specially, and/or discarded racks available free from almost any drug or variety store. One or more students will normally volunteer to serve as class librarian, checking materials out and keeping displays in order.

Obtaining a variety of attractive materials is another problem, one which is sometimes discouraging. Nevertheless, with a lot of commitment and a little persuasiveness most teachers can begin a modest classroom collection and add to it steadily. Many mathematics sourcebooks suggest games, puzzles, or other materials which can be constructed by teachers or students. In addition, there is a rather large number of appealing mathematical materials available commercially. Appendix D contains a list of such materials. The

appendix classifies materials under Early Grades, Middle Grades, and Upper Grades. Within each level, the materials have been clustered as representing Biography and History, Games and Puzzles, or Skills and Concepts. The materials are listed as being particularly appealing to students at the various grade levels. However, no guarantee is offered as to their intrinsic appeal. Some teachers or students may find some of the listed materials distinctly unappealing: too difficult, irrelevant, or unsatisfactory for some other reason. Individual appeal, like those factors that inspire voluntary reading, is an extremely elusive quantity. Hopefully, at least some of the listed materials will be welcome in almost any mathematics classroom.

- **Summary**

The suggestions in this chapter for increasing the amount of voluntary mathematical reading by students are not guarantees for drastic or immediate change in attitudes of students toward mathematics or reading. However, ensuring student success and providing a variety of appealing written materials are teacher behaviours which help students feel more comfortable in mathematics, with the result that teachers feel better about their work. When this happens, the classroom atmosphere is likely to improve perceptibly, even dramatically.

Chapter 9

ASSESSING STUDENT ABILITIES

This topic was deliberately left until last for two reasons. First, it is very tempting to administer lengthy tests to our students without any clear idea of how we intend to use the results. The preceding chapters in this volume should suggest means of providing guidance in mathematical reading when that need is discovered through assessment. Second, many sensitive teachers find that their daily association with students in the mathematics classroom provides them with more information concerning student success than any number of formal tests or inventories. An exception, of course, is the attempt to determine during the first days of a new school year student ability in using the mathematical reading process described in Chapter 2.

• Content Versus Process

From the beginning of this volume, a distinction has been made between mathematical *content* (the essential ideas, understandings, or concepts of the curriculum) and mathematical reading-learning *process* (the acquisition, manipulation, and use of mathematical content in reading and learning). Hence, it should not surprise the reader to find the same distinction made in the present topic—assessment. Although assessment of students' abilities in mathematics can (and should) be performed at the beginning, during, and at the end of instruction, the type of assessment described here is designed to illuminate the students' abilities with regard to *process*, rather than content. In other words, this chapter contains ways to measure the student's ability to process information in mathematics, rather than the amount of information he has acquired. Of course, tests of content are useful and necessary in mathematics classrooms; however, this chapter deals only with means of observing the reading-learning process. For purposes of clarification, examine the following sample of a *content* test in mathematics.

Percentage

Part I Check the statements below that are true.

____1. 4 = 4%

____2. To find the percentage, you multiply the rate times the base.

Part II Complete each statement by filling in the blank.

1. Percent means _____.
2. Wool and rayon socks are advertised as 50% rayon. This means that the socks are _____ wool.

Part III Express as a common fraction or decimal.

1. 40% = _____
2. $12\frac{1}{2}\%$ = _____

Part IV Solve the following problems for rate, base, or percentage.

1. 68% of 563 = _____
2. 5 = _____% of 23

Part V Use your knowledge of percents to solve these problems.

1. Which is the better score, three out of ten or four out of twelve?
2. Mary bought a $5.00 sweater for $4.00. What was the rate of discount?

The preceding test sample attempts to examine the degree to which students have mastered the concepts and computational skills in percentage, rather than the process of reading and/or learning. To repeat, this is *not* the type of assessment being discussed here. Further examination of this chapter will clarify this point by providing specific examples of process assessment.

- **Sequenced Questions**

One of the best ways to assess students' reading abilities in mathematics is through classroom use of sets of questions designed to represent the levels of the reading-learning process presented earlier in this volume. Detailed examples of sequenced questions appear in Chapters 2 and 7. It should be noted that sets of questions such as these are deliberately designed to represent all four levels of the process—perceiving symbols, attaching literal meaning, analyzing relationships, and solving problems. Moreover, the questions are based on text material (preferably an actual reading assignment) selected for use in that class, although not material already covered in instruction. Most importantly, the questions must demand that the students read to be successful, that is, they should not be a test of information already acquired (in other words, a test of content). It should be obvious that the purpose of administering sequenced questions as assessment differs from our recommendations in earlier chapters. The earlier chapters dealt with the use of questions as *guidance*, in order to help ensure student success. The present purpose, *assessment*, attempts to discover at which levels students succeed and at which levels they have difficulty. It implies that teacher assistance would be offered less readily, or not at all. It implies that the teacher would record carefully each student's performance in terms of written work and other behaviour such as visual difficulty or extreme anxiety. In summary, the classroom use of questions

sequenced deliberately to represent the various levels of reading-learning demanded of students can provide the observant teacher with useful tentative information on student abilities regarding required mathematical reading tasks.

- **Informal Reading Inventory**

As with sequenced questions, an informal reading inventory is constructed and administered to ascertain students' mastery of reading process, rather than course content. Like question sequences, it is based on classroom text material, and administered with an eye to written performance as well as other observable test behaviour. However, rather than a sequence of questions on a certain common topic or assignment, the informal inventory consists of several questions designed to test each of a number of reading skills judged by the teacher to be important to the required reading tasks in his classroom. A list of such skills is included as Appendix E. The following inventory is taken from *Evaluating Reading and Study Skills in the Secondary Classroom*, by Ruth Viox (IRA, 1968). Note that the teacher has selected four skills judged essential to mathematical reading in that classroom, constructed several items to test each skill, and designed each item so that textual reading is required.

Mathematics: Group Reading Inventory

Sample Sample Sample

Parts of Textbook

Directions: Referring to your textbook, answer the following questions.

1. How recent is the book?
2. How many chapters does the book contain?
3. On what page does Chapter 7 begin?
4. Within each chapter, the book is divided into what?
5. On what page does the discussion of "Polynomial Functions" begin?
6. On what page will you find information on complex polynomials?
7. Is there a glossary? If so, on what page does it begin?
8. What reference tables are included in the book?

Knowledge of Symbols

Directions: Each of the following symbols is often used in place of words. Match each symbol with its correct meaning.

1. $\neq$	a. is greather than
2. $>$	b. a set
3. $\leq$	c. is less than or equal to
4. $\in$	d. is not equal to
5. $\{\}$	e. is an element of
	f. equals

Analysis of Word Problems

Directions: Make three columns on your answer paper. Label the three columns "Essential Facts," "Nonessential Related Facts," and "Unrelated Facts." For each problem below, classify each underlined segment under the most appropriate heading.

1. A boy, who earns $20.00 weekly from a part-time job, decides to spend up to $24.00 for records and paperbacks. If paperbacks cost $2.00 each and records cost $3.00 each, use a system of inequalities to investigate the possibilities.
2. Last month Nan spent 96¢ for bus rides at 8¢ a ride. Her grandmother earns $140.00 per week. How many rides on the bus did Nan pay for?

Solving Word Problems

Directions: Solve each problem below. Show your work. Label all answers.

1. Mr. Hill wants to paint his kitchen with two coats of paint. It is 16 feet long and 12 feet wide. He bought 2 gallons of paint at $5.55 per gallon. What did the paint cost?
2. Fifty-nine pupils went to Brookville by bus. Sixteen had to stand. How many were seated?

• Cloze Procedure

The cloze is an assessment procedure whereby every fifth (or eighth, or tenth) word is deleted from a section of text, with the student being required to complete each blank with the word that makes the most sense, given that context. The results give the teacher some idea of the general level of a student's comprehension of that text material, rather than information regarding the student's use of specific skills. Here is a detailed set of directions for constructing, administering, and interpreting a cloze test, followed by an actual cloze test constructed for use in assessing mathematical reading.

Directions

1. Select approximately 260 running words from required text material.
2. Print the first sentence in its entirety, unmutilated.
3. Select, at random, one of the next 5 words (i.e., one of the first 5 words in the second sentence). Delete this word and replace it with a blank of standard length. Continue to delete every fifth word until you have 50 blanks. End that sentence. Follow with a complete, unmutilated sentence.
4. Since most students will find the cloze a new experience, it is important to explain the purpose of the test, and to precede its administration by one or two similar very brief and easy exercises, completed with teacher guidance and/or peer collaboration. Administration rarely takes more than 30 minutes.
5. Since the test is not to be graded or returned to the students, the easiest means of scoring is to avoid the search for synonyms. Rather, mark as

correct only those words or symbols which are *exact* replacements according to the original text material. Multiply each correct replacement by two to arrive at percent correct.

6. Research has shown the cloze to be a valid and reliable measure of reading comprehension. As with any test, however, your interpretation of the scores is most important. The research suggests that cloze scores of less than 30 to 35 percent are likely to indicate inadequate comprehension, while scores of greater than 55 to 60 percent are likely to indicate very high comprehension of the text in question.
7. Perhaps the best way to interpret your cloze scores, however, is to organize them in a simple frequency distribution, i.e., arrange the scores in order to show that so many kids got 10 percent, so many got 12 percent, so many got 14 percent, and so on. This kind of organization will give a more graphic picture of how well individuals and groups in a particular class comprehended the text.

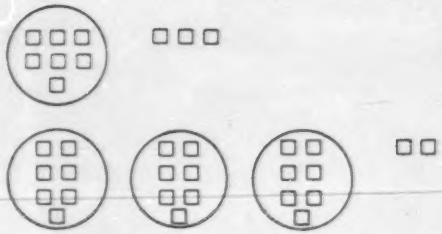
Note: In constructing the cloze test, some teachers use blanks of two standard lengths; for example, fifteen spaces for word symbols and five spaces for numerals and other mathematical symbols. Although this practice tends to give the reader additional clues, its use is defended on the grounds that it preserves the appearance and integrity of mathematical writing, which normally includes numerals and other symbols throughout most printed passages. Also, in order to avoid confusion in identifying blanks (as opposed to lines separating fractions, for example), some teachers print all lines indicating a deletion in red or some other color.

A Typical Mathematics Cloze Test

Our numeral system uses ten as its base. We are so familiar _____ _____ the base-ten system that _____ idea of using a _____ other than ten as _____ base might seem strange. _____, the Celts, who lived _____ Europe more than 2,000 _____ ago, probably used twenty _____ a base. Some Eskimo _____, even now, group and _____ by fives. Let us _____ that we take a _____ into space and land _____ the mythical planet Septus, _____ the inhabitants use seven _____ the base for their _____ system. If we study _____ numeral system used on _____ planet Septus, it will _____ to give us a _____ and more thorough understanding _____ our own base-ten system.

_____ the planet Septus the _____ use square coins. Look _____ the coins shown and

_____ how they are grouped _____ sevens. Here we see _____ group of seven and _____ more. The numeral that _____ this is $13_{(seven)}$. The _____ $13_{(seven)}$ means "one group _____ seven, and three ones."



_____ we see three groups _____ seven squares each, and _____ extra squares. The numeral _____ indicates this is written _____. The numeral $32_{(seven)}$ means "_____ groups of seven, and _____ ones."

In our modern _____ system we do not _____ to write the base _____ everybody understands that the _____ is ten. For example, _____ the decimal system the _____ "27" means two groups _____ ten, and seven ones, _____ $27 = (2 \times 10) +$ _____. However, when we use _____ other than ten, we _____ indicate the base that _____ are using.

In our decimal system we use the ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

(From *Mathematics: A Modern Approach* by M. Peters & W. Schaff, p. 41. Copyright 1971 by Litton Educational Publishing, Inc. Reprinted by permission of Van Nostrand Reinhold Company.)

Note: The key to this test is included as Appendix F.

It is important to note that cloze scores are not to be interpreted as analogous to the various levels of reading as described in Chapter 2. In other words, high scores on the cloze test, while indicating probable high general comprehension of the passage in question, do not indicate the ability to perform at high levels in the model (e.g., problem solution). Nor do low scores necessarily indicate that a student can function only at low levels (e.g., perceiving symbols). According to the definition of reading used throughout this volume, since symbol perception is requisite to all comprehension, it is possible that a low score on the cloze test may indicate the inability to perceive symbols accurately. Therefore, some teachers follow the cloze test with a simple test of pronunciation for only those students whose scores on the cloze are very low (i.e., below 30 to 35 percent).

- **Perception Test**

A test of symbol perception is administered only on an individual basis, while the rest of the students are otherwise occupied. It consists of two lists of symbols, which the student is asked to pronounce while the teacher notes mistakes on another copy. Although any mathematics teacher can compile similar lists, the following is an example which can be used. Notice that List 1 contains common mathematics symbols progressing from very easy common words to more difficult and complex symbols. List 2, on the other hand, contains symbols that are for the most part nonsense words requiring the reader to analyze the word carefully, seeking out structural familiarities in order to achieve correct pronunciation. The purpose of the two lists is to gain information regarding a student's ability to pronounce or remember words commonly appearing in mathematical reading (List 1), as well as his ability to attack completely unfamiliar words in a methodical manner (List 2).

Word Perception—Math

Name_____ Date_____

List 1	*List 2*
add	numberinch
hundred	gridfast
set	supercircum
problem	pervertical
subtract	monogon
thousand	pentangle
column	factorize
group	misinfinity
space	
equally	
symbol	
distance	
opposite	
pentagon	
altogether	
diagonal	
parentheses	
commutative	
metric	
symmetrical	

• Organizing the Information

Needless to say, assessment of student reading ability by any of the above means should not be attempted if the resulting information is not used. Therefore, the information should be organized to allow the teacher to identify both individual and group strengths and plan appropriate instructional activities. A class chart such as the one shown can be very helpful; by reading *across* the chart the teacher can identify individual strengths and weaknesses, while reading in a *vertical* fashion allows the teacher to ascertain whole class performance in a given skill or skill area.

Class Chart—Informal Reading Inventory

Specific Skills or Areas

Name	Parts of Book	Resources	Graphs, Charts	Vocabulary	Explicit Meaning	Relationships		Word Perception	Cloze Score	Comments
Jim A.		√				√			52%	
Rosie B.	√√	√√	√√	√√	√	√√		√	32%	
Chuck C.		√	√	√		√√			46%	
Ann D.	√	√√	a	a	a	√√		√√	20%	
John E.			√√	√	√	√			36%	

Key: No check—Satisfactory performance
Single check—Some difficulty
Double check—Much difficulty
"a"—Absent

• Summary

This chapter has suggested four means of assessing student competencies in mathematical reading, as opposed to accumulated mathematical knowledge: sequenced questions, the informal reading inventory, the cloze procedure, and an individual test of symbol perception. None of these techniques should be employed if the results are not to be used to identify individual or group needs and to provide guidance in required reading, with the goal of success for every student. When needs are identified, the teacher has but to provide guidance according to the suggestions presented in earlier chapters of this volume. Provision of differentiated amounts of guidance based on the techniques described here, or by sensitive observation of any kind, can be an essential first step in individualized instruction that is effective and practical.

AFTERWORD

John Dewey is alleged to have said that the most practical thing for classroom teachers is a good theory. Hopefully, the reader of this volume will have gleaned many immediately useful classroom ideas. More than that, however, the author has attempted to put forth a coherent approach to the teacher's role in teaching reading in the mathematics classroom. The procedures described herein do not constitute *the* approach, only *one* approach, to the problem. No claim is made for comprehensiveness, only coherence. And to the degree that this approach is theoretically sound, it consists of the following principles.

1. Regardless of how well or how poorly students have been taught in earlier grades, a range of knowledge and abilities will be present in any given class at any given level. Individual attention cannot be given to students through materials, administrative grouping procedures, or curriculum innovations. Only the teacher can individualize instruction.

2. The teaching process must always be one of trying to ensure student success in reading and learning; expecting students to know what they come to class to learn represents the exact opposite to the philosophy on which this volume is based.

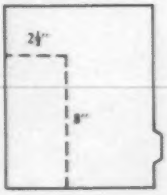
3. Reading is not a subject; mathematics is. Mathematics cannot be studied without reading behaviour as one essential means of subject mastery. Therefore, the teacher who expects his students to grow in mathematics must perforce teach the reading of mathematics as it relates to his course of study. The reading teacher cannot possibly select essential vocabulary and understandings, choose mathematical material, set purposes for reading, and provide the other necessary guidance to ensure student growth in mathematical reading. By the same token, no mathematics teachers should be expected to teach general reading skills, assign practice reading exercises, or take any classroom time whatever for guidance in reading that is not an integral part of his mathematics instruction.

4. The content objectives must always be the determiner of the process levels employed; the teaching of reading skills or levels for their own sake results in a subjugation of mathematics content, a condition that mathematics teachers cannot (and should not) tolerate.

APPENDIX A

Directions for Constructing a Hand Tachistoscope

1. From a manilla folder, cut a strip $2\frac{1}{2}$" by 8".

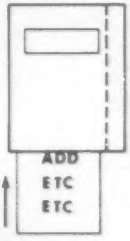


2. In the top fold, cut a window $\frac{1}{4}$" by $1\frac{1}{2}$".

3. Fasten along the vertical edge with staples.

4. On additional 2" by 10" strips, type the words to be exposed. Words should be typed in the center of the strip, triple or quadruple spaced.

5. Tachistoscope is ready to use by inserting strip from bottom and sliding each word past window.



APPENDIX B

Words Appearing Most Frequently in Mathematics Texts

473 words compiled from eight basal arithmetic series—1-3

Frequencies of Usage 1,000+

add
cent
each
equation
how many
hundred
inch
name
number
numeral
one
sentence
set
sum
ten
three
two

Frequencies of Usage 500+

addend
addition
all (in all)
below
by
cost
day
five
from
foot, feet
four
how much
left
less than
line
missing
more
multiplication
part
point
problem
product
six
solve
subtract
subtraction
thousand

Frequencies of Usage 100+

abacus
about
above
after
amount
angle
area
around
before
between
circle
clock
close
column
complete
count
cup
curve
difference
different
digit
dime
divide
division
dollar
dozen
eight
end point
equal
even
even number
factor
figure
first
form (horizontal)
fraction, fractional
frame
fourth
gallon
great
greater than
group
half, halves
hour
inside
join
large
length
less
line segment
long
many
match
measure, measurement
member (of a set)
mile
minute
money
more than
multiply
near
next
nickel
nine
not
number line
number sentence(s)
object
odd
odd number
one-half
open and closed sentence
order
ounce
outside
over
pair
pattern
penny
pint
place
plus
pound
problem solving
quadrilateral
quart
quotient
ray
rectangle
rename
replace
right
row
same
same number
second
segment
seven
shape
side
sign
simple
space
square
subset
table
temperature
third
time (of day)
times
triangle
twenty

Frequencies of Usage 100+ (continued)

unit
week
weight
whole
whole number
most
multiple
yard
zero

Frequencies of Usage 50+

centimeter
change
collection
coordinates
corner
counting numbers
degree(s)
equally
equivalent
estimate
fact
far
few
fifth
fifty
figure
forty
function
graph
half-past
how long
last
ninety
number pair
number pattern
number ray
number word
one-fourth
operation
path
piece
place value
plane
region
relate
remainder
right angle
round
same as
same size
score
separate
seventy
short
sixth
size
small
solution
some
symbol
tall
thermometer
thirty
twelve
undo
union
value
width

Frequencies of Usage 35+

across
as many as
compare
compare(ing) sets
complete the equation
connect
cube
distance
do
double
down
eighty
expanded notation
fifteen
geometry
little
matching
one-to-one/ one-to-one matching
missing factor
one-third
opposite
parallel lines
partition
pentagon
polygon
prime number
regroup
Roman numeral
same length
seventh
shade
short form
sixty
tenth
twice
under
unit of measure
up
which

Frequencies of Usage 25+

addition table
altogether
as much as
associative property
both
cardinal number
center
concave
convex
diagonal
distributive
edge
eighth
eleven
enough
equivalent sets
expanded numeral
fast
fourteen
grid
inequalities
interset
location
ninth
number of a set
open
parentheses
principles
property
sixteen
solving (the) equation(s)
standard numeral
symmetry
take-away
three-fourths
top
total
unnamed
vertical form

Frequencies of Usage Less than 25

addition sign
algorithm
alike
amount product
apart
Arabic numerals
array
as great as
associative
associative law
base
base-ten
big
bottom
braces
calendar
changing the order
circular region
combination

Frequencies of Usage Less than 25 (continued)

comma
common factor
common multiple
common property(ies)
commutative
commutative law
commutative property
compute
connecting points
contain
corner point
cubic
curved line
cylinder
decimal
diameter
distributive property
dividend
divisible by
divisor
eighteen
element
ellipse
empty
empty set
equilateral
equivalence
estimating area
expand
geometric
given set
grouping principle
grouping property
grouping rule
height
hexagon
high
horizontal line
how far
how few
hypoteneuse
identify
identity element
infinite set
inquiry
instead of
intersection of sets
inverse
inverse operation
isosceles
join sets
joining and taking apart
joining equivalent sets
lattice
linear measure
liquid measure
long way
longer than
mathematics
metric
mid-point
middle
minuend
million
minus
model
model factor
model set
named factor
naming numeral
natural order of number
negative numbers
nineteen
nonequivalent sets
nonmetric
none
not equivalent
number family
number object
number of addition
numeration
numeration table
once
one-fifth
one less
one more
one-to-many matching
one-to-one
one-to-one correspondence
only
open and closed figure
order of counting numbers
order of numbers
order principle
ordinal
ordinal concept
ordinal number
parallel
parallelogram
peck
perimeter
place holder
position
plus sign
radius
rearrange
rectangular region
related facts
related equations
related number sentences
relation
remove
repeat
repeated addition
replacement (set)
represent
result
rhombus
sequence
sequence of numbers
separating sets
set of points
seventeen
short way
shorter than
sign of relation
single
six-sevenths
sphere
square unit
straight line
subtrahend
subtraction pattern
such that
surface
symmetrical
thirteen
tiny
triangular region
triangular shape
ton
two-thirds
unequal
unit line segment
unit of time
unknown
unnamed factor
unnamed quotient
vertex (-ices)
vertical
vertical line
volume
word name

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APPENDIX C

Practice Words—Structured Overview

EVEN	ODD
ADDEND	CENTRAL
MEAN	MIDDLE
DIVIDING	MEDIAN
STATISTICS	VALUE
DATA	AVERAGE
TYPICAL	SIZE
FREQUENT	MODE

Directions: Cut along lines to separate words for practice in constructing a structured overview. Use blank slips to add terms necessary to *your* particular arrangement.

APPENDIX D

High Interest Materials for Mathematics Classrooms

Early Grades (K-5)

Biography and History

Brindge, Ruth. *The Story of Gold.* New York: Vanguard Press, 1955.

Larsen, Harold D. *Arithmetic of Long Ago.* Evanston, Illinois: Harper and Row, 1962.

Larsen, Harold D. *The Story of Money.* Evanston, Illinois: Row, Peterson, 1956.

Games and Puzzles

Adler, Irving. *Magic House of Numbers.* New York: John Day, 1957.

Aftermath. Creative Publications, Box 328, Palo Alto, California 94302, 1971.

Buckeye, Ewbank, and Ginther. *A Cloudburst of Math Lab Experiments,* 1971.

Coleman, James S. *Take.* Baltimore, Maryland: Academic Games Associates, 1970.

Degrazia, Joseph. *Math is Fun.* Buchanan, New York: Emerson, 1955.

Dumas, E. *Math Activities for Child Involvement.* Boston: Allyn and Bacon, 1971.

Ewbank, W. *A Downpour of Math Lab Experiments* (5 book series). Birmingham, Michigan: Midwest Publications, 1969–1970.

Golick, Margie. *Deal Me In: The Use of Playing Cards in Teaching and Learning.* Jeffrey Norton Publishers, 1973.

One Minute Game Guide. Palo Alto, California: Fearon, 1968.

Ross, Dorothea. *Pacemaker Games Program.* Belmont, California: Fearon, 1969.

Waugaard and Horlock. *Fun and Games with Mathematics.* Contemporary Ideas, Box 1703, Los Gatos, California 95030, 1972.

Skills and Concepts

Adler, Irving, and Ruth Adler. *Sets and Numbers for the Very Young.* New York: John Day, 1969.

Cohen, Donald. *Geo-Boards.* New York: Walker Educational, 1971.

Computapes (audio cassettes). Chicago, Illinois: Science Research Associates.

Cuisenaire Classroom Kit. New Rochelle, New York: Cuisenaire Company of America, 1968.

Developmental Math Cards. Menlo Park, California: Addison Wesley, 1970.

Feingold, Abraham. *My Workbook: How to Use Numbers.* New York: John Day, 1967.

Harex Multibase Abacus and Activity Cards. Boston: Invicta, 1971.

Hundred Number Board. Boston: Invicta, 1971.

ISM (Investigating School Mathematics). Menlo Park, California: Addison Wesley.

Laboratory Approach to Mathematics. Chicago: Science Research Associates.

Mathematics Laboratory. Cincinnati, Ohio: McCormick-Mathers.

Math Kit. Morristown, New Jersey: Silver-Burdett, 1971.

Multiplication Board. Long Island City, New York: Visionetics, 1969.

Multiplying Machine. Boston: Invicta, 1971.

Napier's Rods and Tablets. Boston: Invicta, 1971.

Problems: Green Set. New York: John Wiley, 1969.

Problems: Red Set. New York: John Wiley, 1969.

Proctor, Charles, and Patricia Johnson. *Computational Skills Development Kit.* Chicago: Science Research Associates, 1965.

Ryder, Ruth T. *Pacemaker Arithmetic Readiness Program, Part A.* Belmont, California: Fearon, 1969.

Stern, Catherine. *Structural Arithmetic, Kit I.* Boston: Houghton-Mifflin, 1965.

Stern, Catherine. *Structural Arithmetic, Kit K.* Boston: Houghton-Mifflin, 1966.

Middle Grades (5–9)

Biography, History, and Fiction

Adler, Irving. *The Story of Numbers, Symbols, and Space.* New York: Golden Press, 1958.

Brindge, Ruth. *The Story of Gold.* New York: Vanguard Press, 1955.

Juster, Norton. *The Phantom Tollbooth.* New York: Random House, 1961.

Larsen, Harold D. *Arithmetic of Long Ago.* Evanston, Illinois: Harper and Row, 1962.

Larsen, Harold D. *The Story of Money.* Evanston, Illinois: Row, Peterson, 1956.

Levinger, Elma E. *Albert Einstein.* New York: Julian Messner, 1949.

Pantheon Story of Mathematics. New York: Random House.

Story of Measures. Evanston, Illinois: Harper and Row.

Games and Puzzles

Adler, Irving. *Magic House of Numbers.* New York: John Day, 1957.

Aftermath. Creative Publications, Box 328, Palo Alto, California 94302, 1971.

Arithmetic Games. Belmont, California: Fearon, 1960.

Degrazia, Joseph. *Math is Fun*. Buchanan, New York: Emerson, 1955.

Fun with Figures. New York: Random House.

Gamow, George. *Puzzle Math*. New York: Viking, 1958.

Gardner, Martin. *Mathematical Puzzles*. New York: Crowell, 1961.

Jokes and Riddles. Evanston, Illinois: Harper and Row.

Larsen, Harold D. *Amusing Problems*. Evanston, Illinois: Row, Peterson, 1956.

Modern Math Games, Activities, and Puzzles. Belmont, California: Fearon, 1970.

Number Magic. Evanston, Illinois: Harper and Row.

Riddell, Mark. *Fun Parade*. Garden City, New York: Doubleday, 1959.

Think Lab. Chicago: Science Research Associates, 1974.

Tripp, Fern. *Star-O and Soc-O*. Fern Tripp, 2035 E. Sierra Way, Dinuba, California 93618.

Skills and Concepts

Adler, Irving. *Giant Golden Book of Mathematics*. New York: Golden Press, 1960.

Adler, Irving. *Time in Your Life*. New York: John Day, 1955.

Anderson, R. W. *Romping through Mathematics*. New York: Knopf, 1947.

Bendick, Jeanne. *Take a Number*. Whittlesey, 1961.

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Buswell, Margaret M. *Counting by Countries*. Evanston, Illinois: Row, Peterson, 1960.

Continuous Progress Laboratory. Palo Alto, California: Educational Progress Company, 1970.

Cooke, David C. *How Money is Made*. New York: Dodd, Mead, 1962.

Ewbank, William A. *A Downpour of Math Lab Experiments* (5 book series). Birmingham, Michigan: Midwest Publications, 1969–1970.

Experiences in Mathematical Ideas, Volume I. Washington, D.C.: National Council of Teachers of Mathematics, 1970.

Experiences in Mathematical Ideas, Volume II. Washington, D.C.: National Council of Teachers of Mathematics, 1970.

Guide to Adventure in Mathematics for the 70s (activity cards). Langley, Bucks: St. Paul Publications.

Highland, E. H. *How and Why Wonder Book of Mathematics*. New York: Grossett, 1961.

Hogben, Lancelot. *Wonderful World of Mathematics*. Garden City, 1955.

Hughes, Toni. *How to Make Shapes in Space*. New York: E. P. Dutton, 1955.

I Can Learn About Calculators and Computers. New York: John Day.

Independent Exploration Material (6 kit series). Danbury, Connecticut: Madison Project/Math Media, 1971.

Juster, Norton. *The Dot and the Line*. New York: Random House, 1963.

Kahn, Charles H., Sylvia K. Herring, and Robert Tong. *Measure Up*. Palo Alto, California: Fearon, 1968.

Kahn, Charles H., et al. *Money Makes Sense*. Palo Alto, California: Fearon, 1960.

Kahn, Charles H., et al. *Using Dollars and Sense*. Palo Alto, California: Fearon, 1963.

Kenyon, Raymond G. *Calculators and Computers*. New York: Harper and Brothers, 1961.

Larsen, Harold D. *Excursions in Arithmetic*. Evanston, Illinois: Row, Peterson, 1956.

Larsen, Harold D. *Faster and Faster*. Evanston, Illinois: Harper and Row, 1962.

Larsen, Harold D. *Magic Squares*. Evanston, Illinois: Row, Peterson, 1959.

Larsen, Harold D. *Roman Numbers*. Evanston, Illinois: Harper and Row, 1960.

O'Neill, Mary. *Take a Number*. Garden City, New York: Doubleday, 1968.

Problems: Green Set. New York: John Wiley, 1969.

Problems: Red Set. New York: John Wiley, 1969.

Russell, Salveig P. *Lines and Shapes*. New York: Henry Z. Walck, 1965.

Tannenbaum, Beulah, and Myra Stillman. *Understanding Time*. New York: McGraw-Hill, 1958.

Using Modern Mathematics. Chicago: Society for Visual Education.

Wiley, Bertha M. *Time and Telling Time*. Palo Alto, California: Fearon, 1967.

Upper Grades (9–12+)

Biography and History

Bell, E. T. *Men of Mathematics*. New York: Simon & Schuster, 1937.

Glenn, W. H. *Invitation to Mathematics*. New York: Doubleday, 1962.

Games and Puzzles

Aftermath. Creative Publications, Box 328, Palo Alto, California 94302, 1971.

Allen, Lyman E. *Equations*. New York: Holt, Rinehart and Winston, 1969.

Allen, Lyman E. *On Sets*. New York: Holt, Rinehart and Winston, 1969.

Allen, Robert W. *Basis*. New York: Holt, Rinehart and Winston, 1969.

Budgets and Taxes. Middletown, Connecticut 06457: Educational Ventures.

Dudeney, H. E. *Amusements in Mathematics*. New York: Dover Publications, 1958.

Euclid. Birmingham, Michigan: Midwest Publications, 1969.

Fadiman, Clifton. *Fantasia Mathematica*. New York: Simon & Schuster, 1958.

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Hunter, J. A. H. *Figures: More Fun with Figures*. New York: Oxford University Press, 1958.

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Mott-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Philadelphia, Pennsylvania: New Home Library, Blakiston Company, 1946.

Pink Pebbles. Middletown, Connecticut: Educational Ventures, 06457.

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Neely, H. M. *Triangles: Getting Ready for Trigonometry*. New York: Crowell, 1962.

Smith, G. O. *Mathematics, the Language of Science*. New York: Putnam, 1961.

Steinhaus, Hugo. *Mathematical Snapshots*. New York: Oxford, 1960.

Weiland, Robert G., et al. *Banking*. Boulder, Colorado: Pruett, 1965.

Weiland, Robert G., et al. *Buying*. Boulder, Colorado: Pruett, 1963.

Weiland, Robert G., et al. *Credit, Loans, and Taxes*. Boulder, Colorado: Pruett, 1967.

Weiland, Robert G., et al. *Wages and Budgets*. Boulder, Colorado: Pruett, 1964.

APPENDIX E

Reading-Study Skills for Mathematics

I. Skills relating to the problem situation
   A. Grasping the problem as a whole
   B. Answering the question, "What are you to find?"
   C. Determining what facts you are given to work with
   D. Deciding what process to use
   E. Estimating answers
   F. Increasing speed in computation

II. Other skills
   A. Interpreting numerical aspects of maps
   B. Reading angles
   C. Reading graphs
   D. Reading formulas
   E. Reading statistical tables in and out of context
   F. Summarizing information for several problems
   G. Recognizing length and relationship of time periods
   H. Charting information needed in solving problems
   J. Mastering technical vocabulary
   K. Noting the concise and concentrated nature of the language of mathematics

APPENDIX F

Cloze Test Key

1. with
2. the
3. numeral
4. a
5. Yet
6. in
7. years
8. as
9. tribes
10. count
11. suppose
12. trip
13. on
14. where
15. as
16. numeral
17. the
18. the
19. help
20. deeper
21. of
22. On
23. natives
24. at
25. note
26. by
27. one
28. three
29. indicates
30. numeral
31. of
32. Here
33. of
34. two
35. that
36. $32_{(seven)}$
37. three
38. two
39. decimal
40. need
41. because
42. base
43. in
44. numeral
45. of
46. or
47. 7
48. bases
49. must
50. we

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